THE DARIAN, UTOPIAN, AND ZUBRIN CALENDARS AND SEASONAL DISSONANCE (PART ONE)

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ABSTRACT

Of the many proposals for a Mars calendar, the Darian (Gangale, 1998), Utopian (Moss, 2000), and Zubrin (Zubrin, 1996) are among the most notable. The first two use 24 nearly uniform months to fill a tropical Martian year that begins at the vernal equinox. The Zubrin calendar also uses a tropical year that starts at the vernal equinox but uses 12 nonuniform months. The two uniform calendars have been criticized for choosing the vernal equinox as their defining point because it requires more intercalation changes than other possible choices. The Zubrin calendar has been criticized for the use of unequal months, an issue that complicates accounting and reporting in our current Common Era calendar.

While the intercalation complaint against the two uniform calendars has merit, a more significant issue has gone unnoticed. The Zubrin criticism also has merit, but this misses both the elegant simplicity of this calendar and its central flaw. Ironically, upon deeper consideration, all three calendars suffer from the same inconvenient fact. The seasons on Mars are neither uniform nor static and no calendar can perfectly conform to them over time.

This paper defines and discusses ‘seasonal dissonance’, the unavoidable conflict between each of these calendars and the evolving seasons. It proposes a metric, ‘date dissonance’, that approximates seasonal dissonance, and describes a method using that metric to minimize the conflict. The results recommend certain tropical events for both the first-of-year and the start of the epoch.

INTRODUCTION

What is missing from many Mars calendar proposals is a chain of reasoning that begins with a simple, accepted definition of a calendar and produces an optimized design. Such a method would involve the following steps. The first paper in this series executes the first five steps.

1. Define a calendar.
2. Determine what characteristic(s) make a good calendar.
3. Determine metric(s) to measure how well a calendar conforms to these characteristic(s).
4. Apply the metric(s) to existing calendar proposals for comparison purposes.
5. Use the metric(s) to improve calendar design.
6. Determine more sophisticated characteristic(s) and repeat from step 3.

ANALYSIS

Criteria for a Calendar

The Meriam-Webster Dictionary defines ‘calendar’ (Merriam-Webster, n.d.) as “a system for fixing the beginning, length, and divisions of the civil year and arranging days and longer divisions of time (such as weeks and months) in definite order.” While this definition accurately describes the final product, it gives no hint as to the principals one might use to design a calendar.

The online Encyclopedia Britannica (Encyclopedia Britannica, n.d.) states of the calendar “It is essential, too, for any civilization that needs to measure periods for agricultural, business, domestic, or other reasons”. This is the critical factor in calendar design. A calendar is developed by and for a civilization or society to track and anticipate cyclic conditions that are important to it. There are several points implicit in this definition. First, the calendar is an ordered collection of time units that conform to cyclic changes in the environment. These changes may involve fluctuations in the temperature due to the tilt of the planet’s axis or its varying distance from the sun or the differences in the length of daylight over the year. Second, the involved cycles must be useful to the society that uses it. The traditional Islamic calendar is a lunar calendar whose homeland has nearly indistinguishable seasons but obvious phases of the moon. The Roman calendar was solar, devised where the seasons are important for agriculture. The Hebrew calendar is lunisolar, incorporating elements of both.

Note that the definition of a calendar does not explicitly require uniformity. Experience with the nonuniform months of the Common Era calendar has shown that nonuniformity presents considerable accounting difficulty for businesses and government. Several attempts over more than a century have been made to make the calendar more uniform, but each has failed for multiple reasons, often involving the momentum of tradition. However, the arch of history shows a tendency toward standardization and uniformity in all measures. The move to the metric system is the most notable example. This paper will consider uniformity to be part of the definition of a calendar but recognize its weaker provenance.

Explicitly, a calendar is a collection of ordered, preferably uniform time units that conform to the cyclic environmental factors that are important to the society that uses it. The best calendar for a society is the one that conforms to relevant cyclic environmental factors over its useful lifespan. In a mathematical sense, since time is linear, a calendar can be understood as a linear approximation of cyclic factors over some time duration.

Background

There are two features of Mars that determine the environmental cycles. Each one affects the light that falls on the planet and the resulting temperate changes that take place over the Martian year (termed in this paper a ‘yare’ [i]). Each feature also determines a coordinate system, or frame, that will be important for the remainder of this paper.

The first feature involves the tilt of Mars' pole relative to the plane of its orbit (obliquity). This factor is, of course, the major determinant of seasons on Earth. Mars' obliquity is currently a bit larger than Earth's at about 25° compared to 23.5°, but it does oscillate about ten degrees in either direction over tens of millennia. The point on the orbit where the north pole is tilted toward the sun has been called the 'summer solstice'. In order to avoid hemisphere bias, the newer terminology is 'northern solstice'. This paper adopts the new terminology. The northward (previously 'vernal' or 'spring') equinox is the point on the orbit where the tilt of the planet is orthogonal to the sun. A measurement from the sun to this point represents the 0° marker of the solar longitude. Mars' poles wobble in much the same way as Earth's. This manifests as a slow rotation of the solar longitude opposite the direction Mars orbits. As this happens, each solstice and equinox rigidly maintains its relative angular separation, acting as a rotating frame, the tropical frame. See Figure 1.

The passage of Mars from any solar longitude on the tropical frame back to that same solar longitude is a tropical yare. As explained below, the length of a tropical yare depends on the solar longitude chosen. The average of all possible choices is the mean tropical yare.

The second feature is the oblate shape of Mars' orbit (eccentricity). This factor is of only minor influence on Earth's seasons, but a major influence on Mars. Mars' nearest approach to the sun (perihelion) is about 17% closer than its farthest point (aphelion). This makes Mars at perihelion about 45% warmer than Mars at aphelion. On Earth the difference is only about 7%. In addition to the large change in solar heating, the eccentricity determines the varying speed of the planet at various points in its orbit. According to Kepler's second law, a line from the sun to a planet sweeps out equal areas in equal time. Therefore, Mars must cover, in a given time, an angle about 45% larger at the perihelion than at the aphelion. A measurement from the sun to the perihelion represents the 0° marker of the true anomaly. Since the perihelion and aphelion are the two 'apsides' of the orbit, this paper refers to this coordinate system as the apsidal frame. The whole ellipse precesses in the same direction as Mars orbits. See Figure 2.

The passage of Mars from any true anomaly on the apsidal frame back to that same true anomaly is an apsidal or anomalistic yare. The length of an apsidal yare generally does not depend on the true anomaly chosen. Apsidal yares may vary due to a combination of the true anomaly and changes to the eccentricity, but for the purposes of this paper there is only one apsidal yare.

In addition to these two coordinate frames, there is another very important coordinate that is simply a mathematical fiction. The mean anomaly is the conceptual angle from the perihelion which a planet would be if its orbit was circular. See Figure 3.

There is a third type of yare that will not play any part in this paper. The sidereal yare is the passage of Mars from some point on the background of stars as seen from the sun back to that same point. Since the inclination of the orbit of Mars may change, that point may not be exactly reached; but these details do not matter here.

Each point on the tropical frame slowly rotates relative to the apsidal frame at an effective constant angular rate in a direction opposite that in which the planet revolves about the sun. This is referred to as the relative precession and a full cycle as a precession cycle. The term uber-yare[ii] is substituted in this paper to keep it compatible with other papers in this series. Estimates for the duration of the uber-yare range from about 50,000 to 57,000 Common Era years. This paper uses the longer estimate for consistency with other constants used here. That duration is about 30,392 yares.

To sum up the effects of these two factors; the tropical frame determines **where** the seasons occur, and the apsidal frame then determines **when** the seasons occur.

A value that will be important in this paper is the time difference between when a tropical event, such as the northward equinox, occurs and when it would occur if Mars had a circular orbit. This is referred to here as the 'equation of date'. It is simply an extension of the equation of time, made manifest in the analemma[iii], but it ignores any terms involving obliquity.

The constants used in equations are defined in Table 3. They were derived from a paper by Michael Allison. (Allison, 1997).

The following equation set represents the 'equation' of date. It can be used to find the mean anomaly from the true anomaly. It is necessary to estimate and iterate to find the true anomaly from the mean anomaly.

Equation 1: Equation of Date

> M (MU) is the mean anomaly. θ (theta) is the true anomaly. E (EPSILON) is called the eccentric anomaly. Δ (DELTA) is the difference between the mean and true anomaly, sometimes called the 'equation of center'. Y_A is the length of the apsidal yare. O (OMICRON) is the 'equation of date'.

$$E = sin^{-1}\left(\frac{\sqrt{1-e^2}\sin(\theta)}{1+e\cos(\theta)}\right)$$

$$M = E - e\sin(E)$$

$$\Delta = (\theta - M)$$

$$O = \frac{Y_A\,\Delta}{2\,\pi}$$

The current values of the equation of date at each point on Mars' current orbit is charted in Figure 4. The symbols presented on the chart represent the four standard seasonal boundaries and four mid-season markers. These symbols are defined in Table 1. This paper uses the Celtic names for the four mid-season markers due to their convenient definition and available symbols.[iv] Lower case is used to distinguish them from the Celtic holidays. They are defined in Table 2.

In Figure 4, one can see that the southern solstice occurs about 29 sols after the perihelion and about six sols after its mean date within the uber-yare. The northward equinox occurs about 183 sols after the perihelion and 19 sols after its mean date. Over the course of one uber-yare the relative precession moves each tropical marker to the left. Eventually each marker will take on each of the values on the curve. The result of this movement is the length of a tropical yare based on any marker constantly changes.

The relative precession causes Mars to return to the same tropical point before it does the same apsidal point. This makes any tropical yare shorter than the singular apsidal yare. Since the speed of Mars changes depending on the apsidal frame, the time it takes to traverse the remaining distance to complete one apsidal revolution varies with the angle from the true anomaly. See Figure 5. The length of the yare can be determined by adding the first derivative of the equation of date to the mean tropical yare.

Equation 2: Length of Tropical Yares by True Anomaly

E'_θ is the first derivative of E in terms of θ. M'_θ is the first derivative of M in terms of θ. Δ'_θ is the first derivative of Δ in terms of θ. Y_m is the length of the mean tropical yare. U_C is the number of apsidal yares in a uber-yare. Y_θ is the length of the specific tropical yare at θ.

$$E'_\theta = \frac{\sqrt{1-e^2}}{1+e\cos(\theta)}$$

$$M'_\theta = E'_\theta(1-e\cos(E))$$

$$\Delta'_\theta = (1-M'_\theta)$$

$$Y_\theta = Y_m + \frac{\Delta'_\theta\, Y_A}{U_C}$$

Although a tropical yare is defined by some point on the tropical frame, the position of this point on the apsidal frame determines how long the tropical yare will be. In the case where the yare-defining point, such as the southern solstice, is near the perihelion; Mars is moving near its fastest speed and there is only a short time between the end of the tropical yare and the completion of an apsidal revolution. Therefore, the tropical yare based on that tropical point is relatively long. When the southern solstice precesses to near the aphelion, Mars will be going near its slowest speed, so the tropical yare will then be relatively short. This means that the length of the tropical yare will change at a predictable rate as the defining point moves along the apsidal frame. It also means that every point on the tropical frame is occurring earlier or later during the yare depending on its point on the apsidal frame. Therefore, the time between any two tropical points varies with the relationship between the tropical and apsidal frames. This causes the duration of the seasons to change.

It is clear that there will be a disagreement between the dates that appear on any calendar with a fixed number of Martian days (sols) in its months, uniform or not, and the varying natural environmental cycles that define a calendar

The Proposed Calendars

Since the three calendars being discussed here are based on the tropical yare, this paper assumes that a tropical yare of some type is the correct type of calendar for Mars. Another paper will be needed to test this assumption.

Zubrin Calendar

The Zubrin calendar divides the tropical yare into 12 months, beginning at the northward equinox. Each month is defined by a 30° arc of solar longitude. This makes the months conform to portions of the tropical seasons. Since Mars varies in speed along its orbit, the months vary greatly in duration. See Table 4. The initial lengths of the months calculated were somewhat different than the Zubrin calendar, probably due to varying sources for the orbital parameters. This should not qualitatively affect the results.

When this author first examined the Zubrin calendar he was willing to dismiss it due not to its obvious nonuniformity but to its transience. It appeared that the lengths of the months were fixed to match the temporary configuration of the Mars orbit. As will be discussed below, this configuration changes in a surprisingly short period. Anchoring the first sol of the yare to the northward equinox only doubles the rate at which the southward equinox moves across the calendar.

As stated above, the Zubrin calendar defines months to span each 30° solar longitude after the northward equinox. However, at this point in time the northward equinox tropical yare circumscribes only 359.998° true anomaly. Therefore, there are only about 29.999° true anomaly in a month.

It was not clear from Dr. Zubrin's paper whether he intended to vary the length of each month as precession advanced. If the lengths of the months do not change, the Zubrin calendar would be obsolete in less than a Martian century.

This author examined the start dates of the months over one full uber-yare at intervals of about 45.23 yares or 85.06 years. See Figure 6. Results indicate that two months will change start date within 50 yares. Since these months were not contiguous, four months must change duration. Every month will have changed duration within 362 yares after the epoch of 2000CE. Clearly, these static months do not meet the strong criteria for a calendar.

Consider the evolving month interpretation. It can be argued that this is simply a calendar that uses intercalation at the month level; a technique used by the Darian calendar to much less extent for leveling quarters. Clearly this interpretation does an excellent job of addressing the stronger criterion within the calendar definition. The Zubrin calendar 'touches base' with the reality of the tropical yare 12 times a yare, while the two uniform calendars touch base only on the first of each yare. Using the angular definition of a month would ensure that this tight relationship exists in perpetuity.

It may be argued that varying the wholly human constructs of months to conform to a real natural cycle is a fair solution. On the other hand, a calendar in which at least one month changes its duration per century introduces a new dimension of nonuniformity that may be termed inconsistency.

Before leaving the Zubrin calendar, it must be stated that there are circumstances under which the Zubrin calendar is much more attractive. If a Mars colony develops that exists solely in orbit, underground, or within less than 30° of the equator, a calendar based on the apsidal yare may be more appropriate. In this case, months based on an equal number of degrees of true anomaly would change only gradually due to slow changes in Mars' eccentricity. These changes would take place much more slowly than the changes discussed above and could probably be ignored. The start of an apsidal yare would require the choice of some point on the apsidal frame and not the tropical frame. There is much to be discussed about this possibility that is beyond the scope of this paper.

Darian and Utopian Calendars

The Darian and Utopian calendars divide the tropical yare into 24 months of roughly equal durations starting at the northward equinox. Some of the months are truncated by one sol to implement intercalation and balance quarters as much as possible.

These calendars certainly address the weaker uniformity portion of the definition of a calendar. The following will examine how well they address the stronger criterion of conforming to cyclic environmental factors that may be important to a Martian society.

Relative Date Dissonance

The online Lexico Dictionary (Lexico Dictionaries | English, n.d.) defines dissonance as "Lack of agreement or harmony between people or things." The two disharmonious elements involved here are the time of a tropical event is 'expected' by a calendar and when it actually occurs. There are two ways of defining 'expected' that determine two types of date dissonance.

The first type of date dissonance is founded on the concept of stored experience. Any person or society studying or living on Mars will experience precession as a slow movement of seasons across the calendar. The early history of this relationship will be permanently associated with the climate and seasons of its time. Initial weather records will record the initial dates of temperatures, daylight duration, and dust events. In order to promote consistent scientific records and a better understanding of the recorded history of Mars, it is important to find a calendar that minimizes the environmental changes that occur for each calendar date throughout an extended time period.

In order to accomplish this, one or more metrics are required that measure relevant changes over time. Since the calendars being discussed here are tropical, this metric must be the dissonance involving some cyclic tropical factor or factors. Two of these metrics are the duration of daylight and insolation. Insolation is the warming of the surface by solar radiation. Both of these are directly related to the obliquity of the planet in relation to the sun. A simple surrogate for the complex calculations involved with these metrics is simply the difference between the date of a tropical event, such as an equinox, occur and when the date it occurred in the first yare experienced. This will be termed ‘relative date dissonance’.

Equation 3: Relative Date Dissonance

> λ_t is the solar longitude of some tropical marker at time t. $\text{♈}_{\theta t}$ is the true anomaly of the northward equinox at time t. θ_t is the true anomaly of the marker at time t. $O_{\theta 0}$ is the equation of date for the marker at the epoch. $O_{\theta t}$ is the equation of date for the marker at time t. $D^r_{\theta t}$ is the relative dissonance at the marker at time t.

$$\theta_t = \lambda_t + \text{♈}_{\theta_t}$$

$$D^r_{\theta_t} = O_{\theta_t} - O_{\theta_0}$$

On Earth, relative date dissonance is negligible. The orbit of Earth is so circular that the seasons vary only by a few days over an entire uber-yare. Anchoring the first of the yare to some tropical marker, such as the northward equinox as done by the Romans, effectively conforms a calendar to the natural tropical cycles. The opposite is true for Mars; anchoring the first of the yare to some tropical marker increases the dissonance between the calendar and the natural cycles.

It is clear there is a much larger relative date dissonance in the uniform calendars that in the Zubrin calendar. In the Zubrin calendar, deviations from the tropical dates occur only during each of the twelve months and are thus much smaller.

If a tropical yare begins at a tropical marker significantly above or below the zero of its equation of date, the other markers will be on dates skewed in the opposite direction from their mean dates. As the uber-yare progresses, these other markers will stay skewed to one side, as shown in Figure 7. This will cause the opposite marker to wander widely across the calendar, 80 sols or more from its original date. Not only is this twice as far as will occur with a mean tropical yare, but this marker would spend about 2/3 of the yare further from its original date than the maximum difference within a mean tropical yare. The separation between the mean date of the opposite marker and the actual date of the seasonal event will peak at about 40 sols. This contradicts the primary purpose of a calendar.

If a tropical yare begins at a tropical marker near the zero of its equation of date, the other markers will be distributed evenly about their mean dates. As the uber-yare progresses, these markers will move across the calendar opposite to the start marker’s motion relative to its equation of date. See Figure 8. The opposite marker will travel just over 40 sols across the calendar to remain much closer to its original date. The separation between the mean date of the opposite marker and the actual date of the seasonal event will peak at about 20 sols.

The above demonstrates that if one selects a tropical yare, the selection of one of the solstices as the beginning of the yare would be preferable to selecting an equinox. The best solution for a tropical calendar is to use the point on the tropical frame currently at the perihelion. This would produce a very symmetric calendar, but this selection has no traditional meaning.

Choosing to use a mean tropical yare would involve starting the yare not at a tropical marker but at the point when the marker would occur if Mars' orbit was circular. That is equivalent to using the equation of date to adjust the start date, or more simply to using the mean anomaly instead of the true anomaly of the marker. Figure 9 and Figure 10 show the results of using the mean tropical yare at two separate markers. This would minimize the maximum and average time between the actual marker and its mean date on the calendar. It is worth noting that the two graphs are nearly identical; using a mean tropical yare makes all tropical markers virtually indistinguishable regarding relative date dissonance.

The above results are reinforced in Figure 11 , which shows the root mean square (RMS) of relative date dissonance for each calendar.

Figure 12 shows the how the RMS evolves over the uber-yare. Note that the northward equinox tropical yare remains competitive with the mean tropical yares for several thousand yares. Figure 13 shows the maximum relative date dissonance for all yares across the uber-yare.

Figure 14 shows which part of the yare contributes to the RMS over the uber-yare, and Figure 15 shows which part of the yare contributes to the maximum over the uber-yare. Clearly, the two tropical yares do not conform to seasonal variations near the middle of the calendar yare.

Figure 16 and Figure 17 show the variation in the dates of seasons over an uber-yare for a tropical and mean tropical yare respectively. Note that the seasonal boundaries for the mean tropical yares more closely match the straight lines of a particular date than the seasonal boundaries of the tropical yares. It is also worth noting that the average date for each seasonal boundary within the mean tropical yares is more clearly on a quarter of a yare.

Central Date Dissonance

While it is the belief of this author that relative date dissonance should be the primary consideration when evaluating a calendar, there is a second type of date dissonance the bears mentioning. One may argue that relative date dissonance should not be based on dates from the first yare of the epoch but on the mean dates to the horizon. This will be termed 'central date dissonance'.

Equation 4: Central Date Dissonance

Θ_0 is the true anomaly of a tropical marker at the epoch. θ_H is the true anomaly of the marker at the lifespan horizon. $O_{\theta 0}$ is the equation of date for the marker at the epoch. $O_{\theta t}$ is the equation of date for the marker at time t. $D^c_{\theta t}$ is the central date dissonance at the marker at time t.

$$D_{\theta_t}^{c} = \mathrm{O}_{\theta_t} - \frac{\int_0^H \mathrm{O}_{\theta_x}\, dx}{\theta_H - \theta_0}$$

When using an entire uber-yare as the lifespan, the equation of date is the same at the beginning and the end except for the effects of eccentricity. These are relatively small over the first full uber-yare of the calendar.[v]

Equation 5: Mean of Central Date Dissonance

$$\frac{\int_{\theta_0}^{\theta_H} \int_0^H \mathrm{O}_{\theta_t}\, dt\, d\theta}{H(\theta_H - \theta_0)} \approx 0$$

Therefore, the following may be used for central date dissonance.

Equation 6: Approximation of Central Date Dissonance

$$D_{\theta_t}^{c} \approx \mathrm{O}_{\theta_t}$$

Although this approximation was developed for a horizon of one uber-yare, it is easy to calculate and a reasonable surrogate for more complicated formulae over shorter lifespans.

Figure 18 and Figure 19 show the central date dissonance for tropical yares at the northward equinox and southern solstice. Note that they are identical except for a phase transition of one quarter an uber-yare. Figure 20 and Figure 21 show the central date dissonance for the same tropical markers. Note that the dissonance is minimized throughout and that the two charts are indistinguishable without close examination of the interior structure.

The overall results of central date dissonance are similar to those of the relative date dissonance, except this dissonance does not recognize the asymmetry issues suffered by the tropical northward equinox.

The Calendar is Mu

There are two options that would introduce both uniformity and consistency into the Zubrin calendar. First; the months could be set to their mean lengths over the uber-yare. This would make the first three months 55 days, the next eight 56 days, and the last sometimes 55 and sometimes 56. This would make the Zubrin calendar a uniform calendar, with the same advantages and about the same date dissonances. Of course, the short months could be moved to smooth the quarters, closing the gap further.

The second option takes advantage of the Zubrin calendar's elegant definition. This can be reworded to state, ”The months have an equal share of the true anomaly between consecutive northward equinoxes.”. The first option would modify this to, “The months have an equal share of the mean anomaly between consecutive northward equinoxes.”. The second option would be stated, “The months have an equal share of the mean anomaly between mean anomalies of the northward equinox.” The advantages of this will be discussed below.

There is one option that would maintain the uniformity of the Darian and Utopian calendars while reducing the date dissonances. Recall that the mean anomaly is simply an angular representation of time. The definition of these calendars could be reformed to state, “The months have an equal share of the mean anomaly between consecutive northward equinoxes.”. This is identical to the first option for the Zubrin calendar. The single option would state, “The months have an equal share of the mean anomaly between the mean anomalies of consecutive northward equinoxes”. This is identical to the second option for the Zubrin calendar.

That final shared definition would reach the optimal compromise between uniformity and environmental conformity. The one feature that some may see as objectional is that the northward equinox would be near, but not always at, the first of the yare. As can be seen in Figure 4, every point on the tropical frame wanders about its mean in a very similar manner as local noon wanders about mean noon in an analemma [iii]. As stated before, the equation of date and the equation of time are identical except for those terms that involve the obliquity. In a critique of the Zubrin calendar Gangale commented on the varying length of the months “…this is exactly the opposite logic from the design of the modern clock.” Using the northward equinox as the start of the yare and using changes in intercalation to maintain that relationships is both qualitatively and quantitively the same as using noon on November 17th as the mean noon on Earth and varying the day to maintain that relationship. It is worth noting here that the required intercalation for these calendars already demand that the start date of a yare is not always exactly at the northward equinox.

Figure 22 displays two generalized Mars calendar with future tropical boundary markers shaded. The larger shaded areas in the tropical calendar do not fully reflect the difference. The tropical markers at the end of the sixth and 18th month will spend more than 40% of the time beyond the maximum reached by their respective markers within mean tropical calendar. As stated before, the tropical marker at the end of the 12th month will spend about 66% of the time beyond the maximum achieved by its counterpart within the mean tropical calendar.

The advantages of these recommended changes are as follows.

1. The above simple definition not only defines the first of the yare and the first of each month; but exactly defines every week, sol, hour, minute, and second.
2. The yare would be the mean tropical yare. There would be only one intercalation domain, barring any unpredicted change in orbital parameters. This increases the consistency implicit in the uniformity criterion.
3. The date dissonance would be reduced to the minimum possible in a uniform calendar. Since date dissonance is only a surrogate for the dissonance of various factors, it remains to be proved that the recommended change produces a minimal result in these factors. This will be addressed in the second paper in this series. Indications are that both seasonal dissonance types will be reduced even further than their respective date dissonances due to an insensitivity to differences in dates in parts of the orbit.

Other advantages do not directly address the two criteria but are worth noting.

4. All dates are treated equally. Each date experiences nearly the same central date dissonance over the course of an uber-yare with minor differences due to changes in eccentricity. Although this paper assumes the northward equinox will be used to calibrate the first day of the yare, any point on the tropical frame could be used.
5. All yares are treated equally. Each yare has nearly the same total central date dissonance with minor differences due to changes in eccentricity. That makes such a calendar epoch insensitive.

A modified calendar could indicate a preference to a particular epoch without actually requiring it. If the northward equinox is the calibrating tropical point, one would expect the epoch to begin when the equation of date for that point is zero. That is also the time when the northward equinox occurs at an apsis. The most recent apsis-northward equinox conjunction was about 5,992 yares (11,270 years) ago. That would be 9,251 BCE. This was near the end of the Younger Dryas on earth. This geological event may have triggered the start of agriculture and would be an excellent zero yare to encompass the history of an expanding human civilization. This is, coincidentally, close to the beginning of the proposed Holocene calendar (Emiliani, 1993); which begins its epoch in 10,000 BCE. Also, in 2013 the International Union of Geological Sciences (IUGS) adopted the year 9,700 BCE as the beginning of the Holocene (post-glacial) epoch (Walker, et al., 2009). This may provide some basis for a future interplanetary epoch.

CONCLUSION

A calendar is a time-linear (uniform) approximation of environmental cycles important to the society that uses it. The Darian and Utopian calendars speak to the uniformity with minimal concern for the cycles that underlie a calendar. The Zubrin calendar addresses the underlying cycles, but with little regard for uniformity.

No calendar can perfectly address both criteria, so a compromise must be made. Any calendar with acceptable uniformity and minimal deviation from important environmental cycles over an extended horizon must be a mean tropical calendar. The northward equinox may still be employed to calibrate the start of the yare, but it will oscillate within about three weeks of the start of the yare plus any intercalation adjustments. Such a calendar would require only one built-in intercalation domain, leaving intercalation changes for unpredicted orbital element variations.

To achieve this, the Darian and Utopian calendars would use the mean, and not the true, anomaly of the northward equinox to calibrate the yare. The Zubrin calendar would have to use the mean, and not the true, anomaly of each twelfth of the yare to determine the lengths of the resulting uniform months.

FURTHER RESEARCH

The Darian and Utopian calendars may achieve lower date dissonance as tropical calendars than mean tropical calendars over a much shorter horizon. The determination of these horizons is left to the calendar proprietors.

Since this paper involved three tropical calendars, neither the sidereal nor apsidal calendar types were discussed. While the sidereal calendar has few proponents and might be relatively easy to dismiss, the apsidal calendar may be dominant for Mars colonization under some circumstances. These will be addressed in a future paper.

Determining the two date dissonance metrics is the first step in conforming a calendar to the natural cycles of a planet. A deeper look involves identifying which cycles are important and how well the calendar conforms to these cycles. The resulting seasonal dissonance metrics promise to illuminate all aspects of the Mars calendar, including the best start date and epoch. This will be addressed in the second paper in this series.

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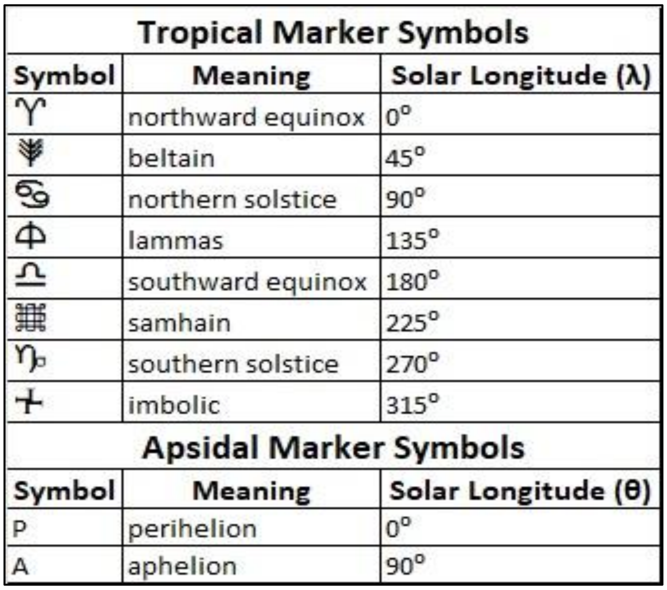
APPENMDIX A: TABLES

Table 1: Chart Symbols



Tropical Marker Symbols		
Symbol	**Meaning**	**Solar Longitude (λ)**
♈	northward equinox	0°
ψ	beltain	45°
♋	northern solstice	90°
Φ	lammas	135°
♎	southward equinox	180°
⌗	samhain	225°
♑	southern solstice	270°
+	imbolic	315°
Apsidal Marker Symbols		
Symbol	**Meaning**	**Solar Longitude (θ)**
P	perihelion	0°
A	aphelion	90°

Table 2: Non-Standard Term Definitions

Term	Meaning
yare	Martian year (alternative: mir)
beltain	45° from the northward equinox
lammas	45° from the northern solstice
imbolic	45° from the southward equinox
samhain	45° from the southern solstice
uber-yare	One cycle of the relative precession between the apsidal and tropical frames, often called the precession cycle

Table 3: Constants Used in Equations

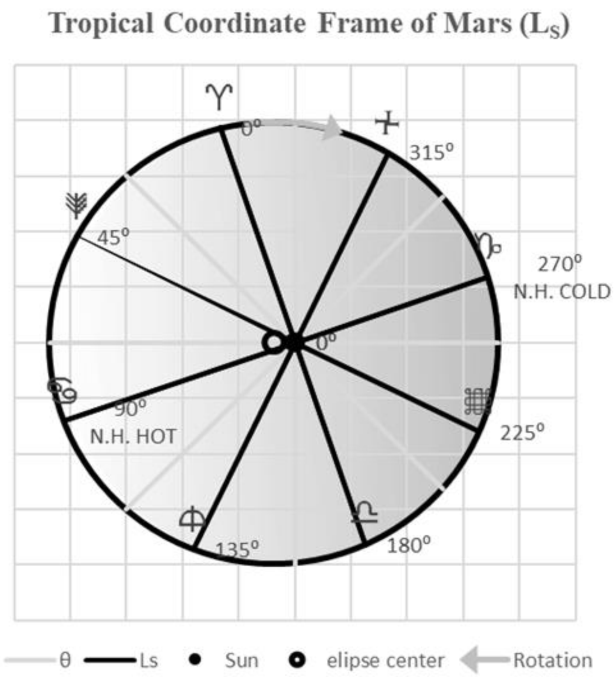
Equation Constants	
Element	**Value**
sols in an apsidal yare	668.61410
sols in a mean tropical yare c2000	668.59210
apsidal yares in an uber-yare (precession cycle)	30,391.55000
northward equinox true anomaly c2000	1.90276

Table 4: Sols Currently in Zubrin Months

Sols in Zubrin Months		
Month	**Zubrin**	**Calculated**
1	61	61
2	65	66
3	66	66
4	65	65
5	60	60
6	54	54
7	50	50
8	47	46
9	46	47
10	48	47
11	51	51
12	56	56

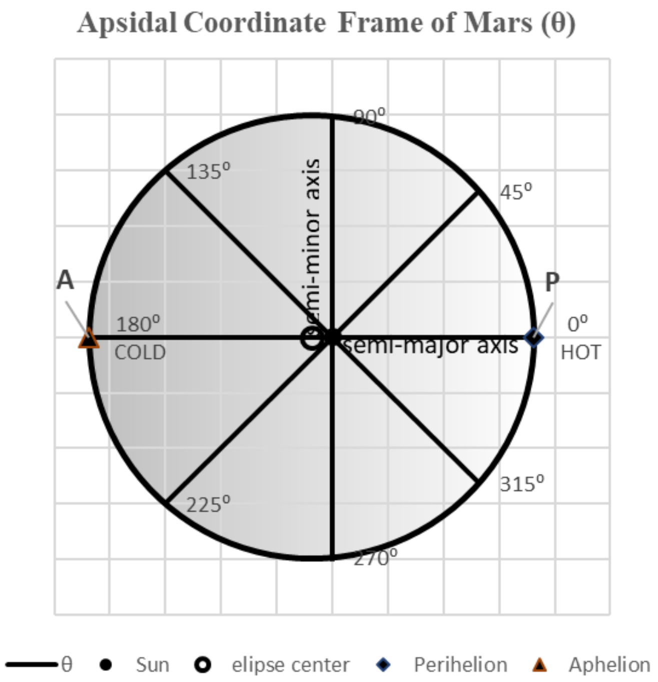
APPENDIX B: FIGURES

Figure 1: Tropical Coordinate Frame



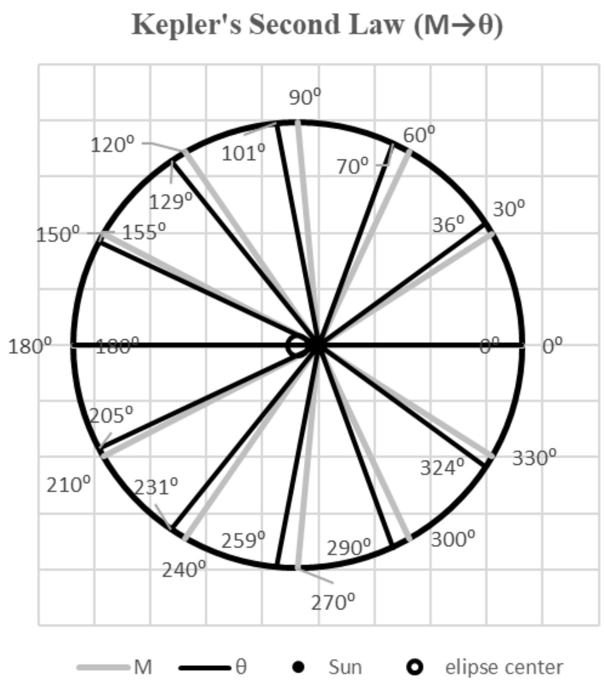
This tropical coordinate frame measures an angle from the northward (vernal) equinox in the direction the planet revolves. This angle is called the solar longitude and is denoted by L_S.
The shading indicates the heat that impacts the northern hemisphere.

Figure 2: Apsidal (Anomalistic Coordinate Frame



The apsidal coordinate frame measures an angle from Mars' nearest approach (perihelion) in the direction Mars revolves. This angle is called the true anomaly or just anomaly and is usually denoted θ (theta) or occasionally ν (nu).
Eccentricity (e) is the distance from the sun to the elliopse center divided by the semi-major axis.
The shading indicates the intewnsityof the solar constant.

Figure 3: Kepler’s Second Law and the Mean Anomaly



Kepler's second law states that a line from the sun to a planet sweeps out equal areas in equal times. Each heavy black line (θ) indicates how far Mars will travel in one eighth of a yare.

The mean anomaly is denoted by M (mu) and indicates where Mars would be if its orbit was circular.

Figure 4: Equation of Date

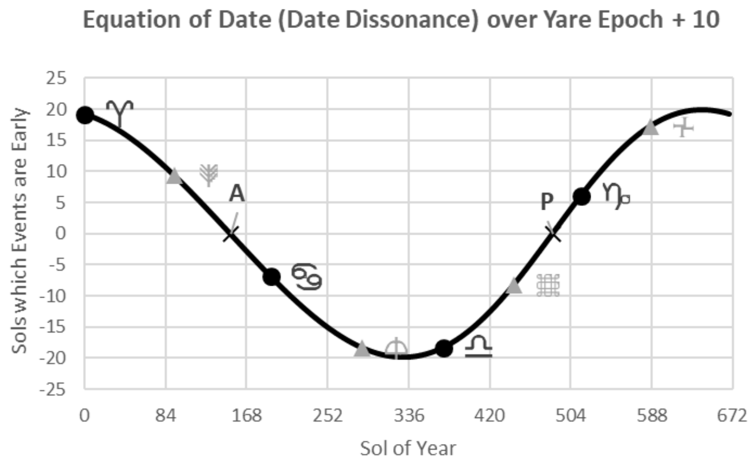


Figure 5: Current Yare Lengths and Intercalation Rates

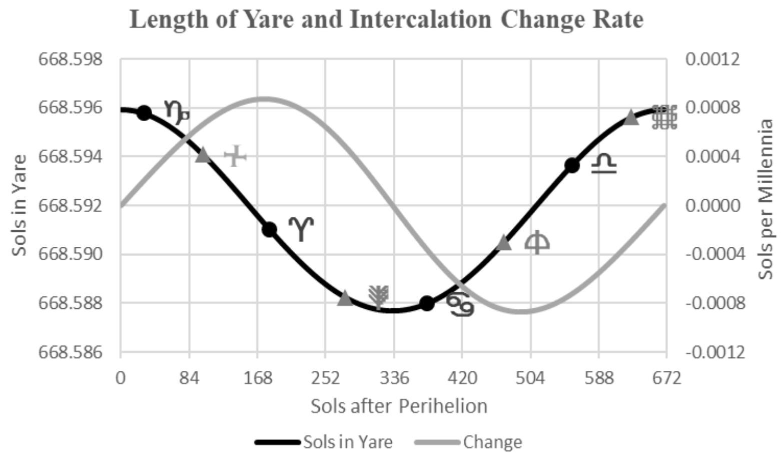


Figure 6: Evolution of the Zubrin Months over Uber-Yare
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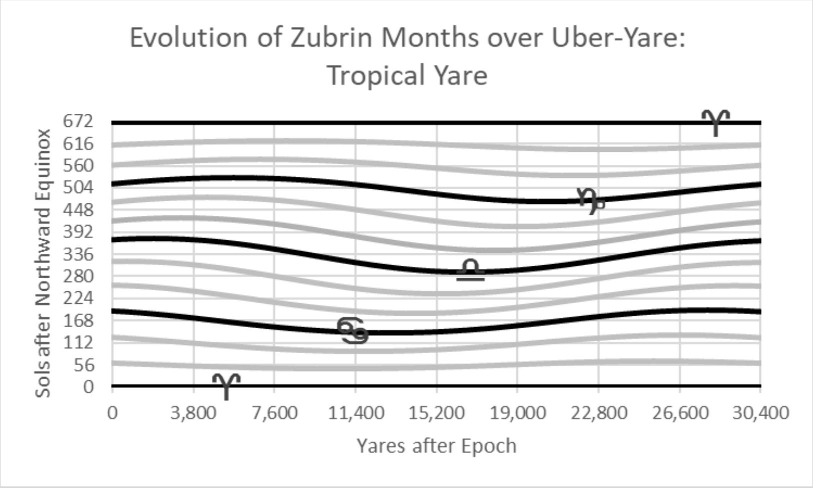

Figure 7: Relative Date Dissonance of Northern Equinox Tropical Calendar over Uber-Yare

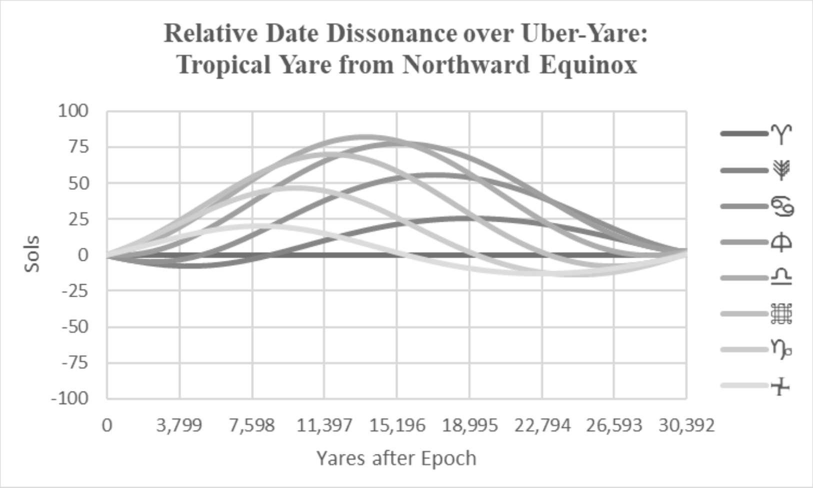

Figure 8: Relative Date Dissonance of Southern Solstice Tropical Calendar over Uber-Yare

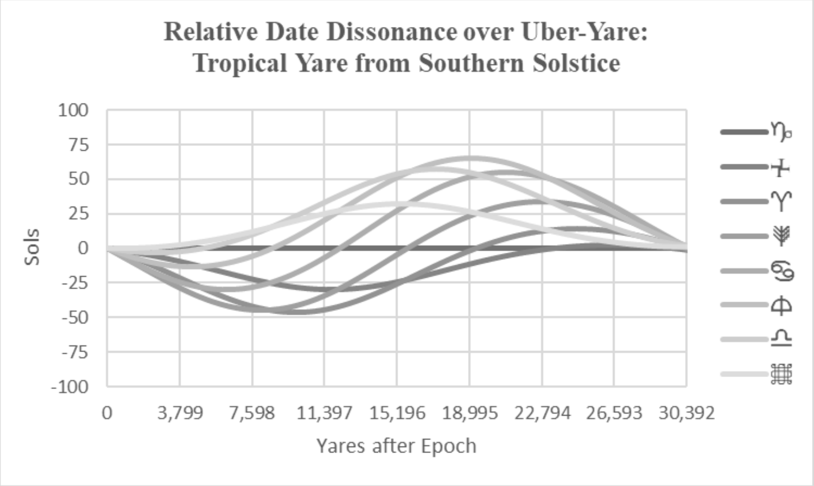

Figure 9: Relative Date Dissonance of Southern Solstice Tropical Calendar over Uber-Yare

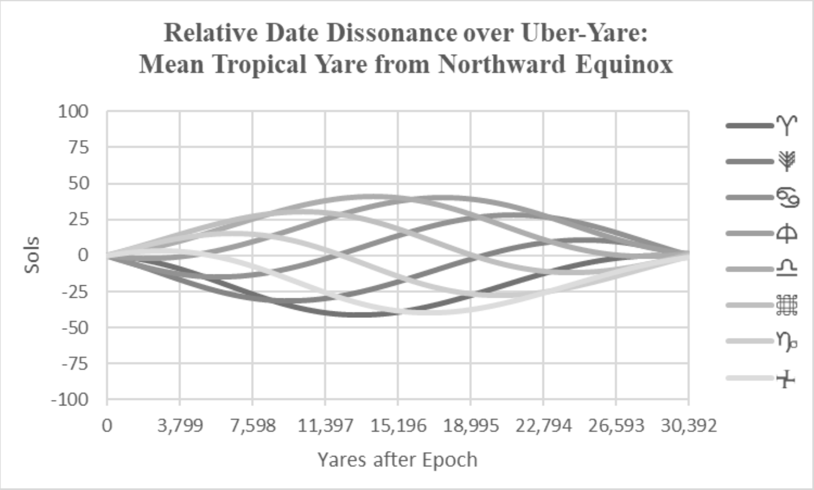

Figure 10: Relative Date Dissonance of Southern Solstice Mean Tropical Calendar over Uber-Yare

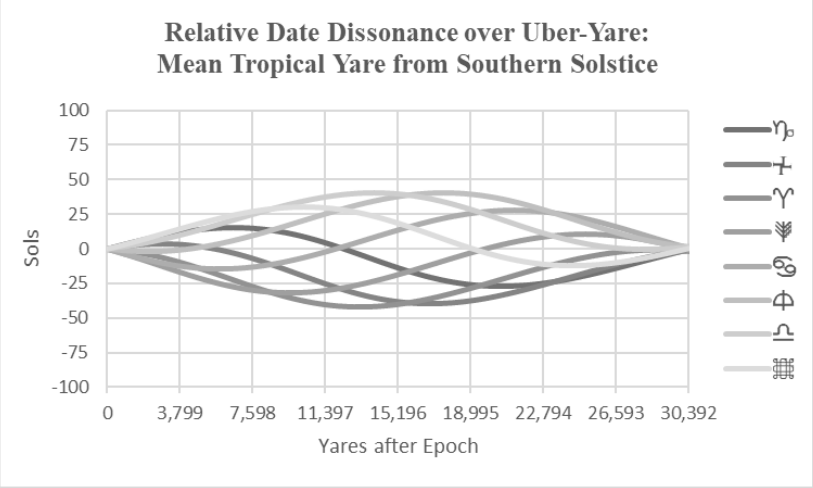

Figure 11: Comparison of Calendars over Uber-Yare

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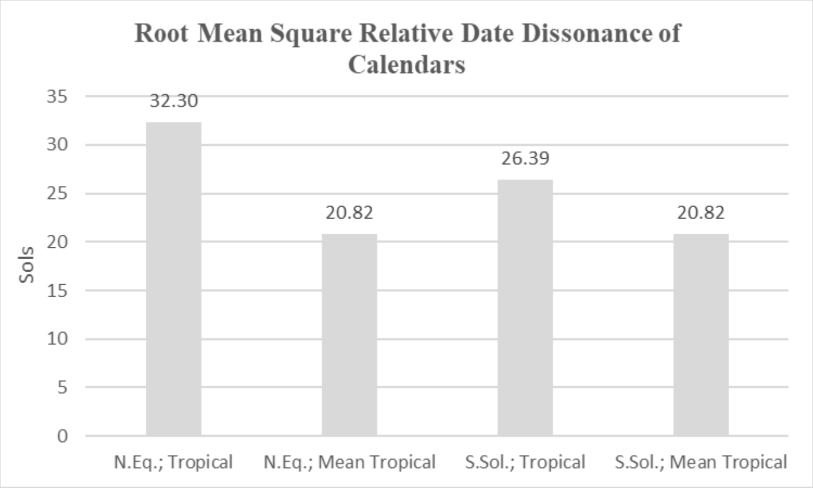

Figure 12: Root Mean Square Relative Date Dissonance over Uber-Yare

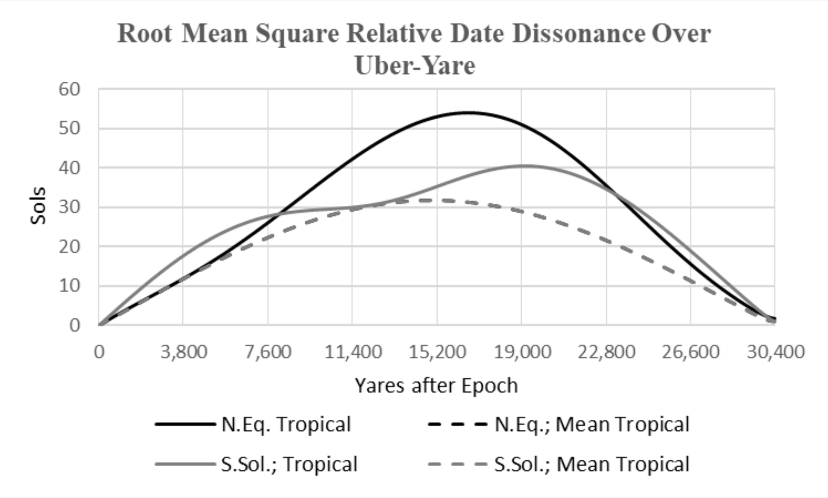

Figure 13: Maximum Relative Date Dissonance over Uber-Yare

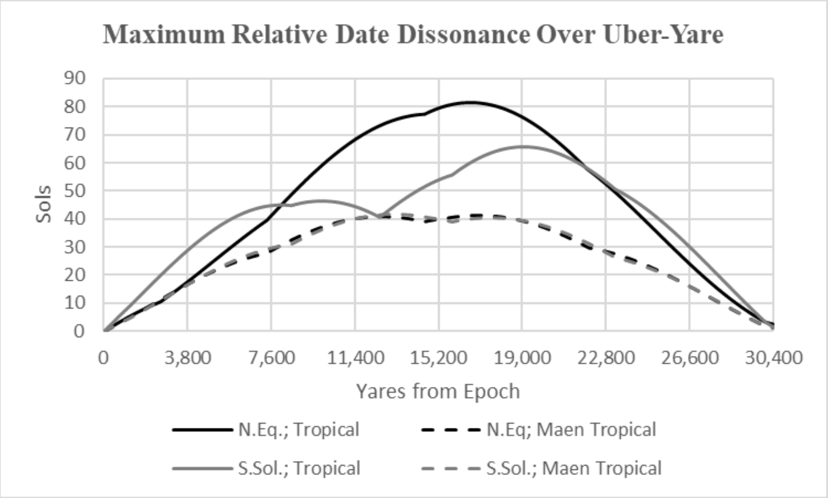

Figure 14: Root Mean Square Relative Date Dissonance over Yare

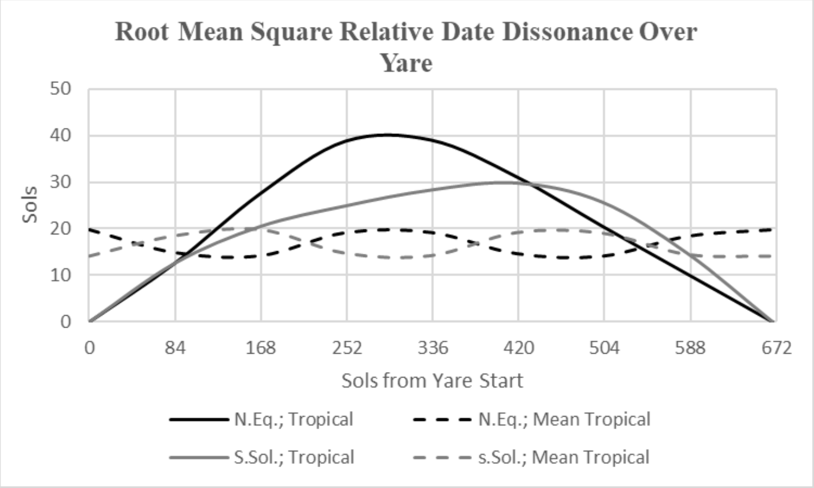

Figure 15: Maximum Relative Date Dissonance over Yare
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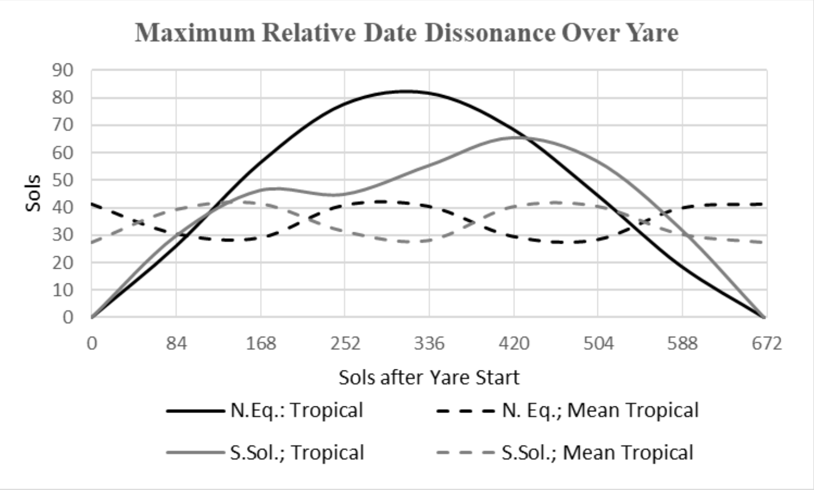


Figure 16: Northern Equinox Tropical Yare Seasonal Variation over Uber-Yare

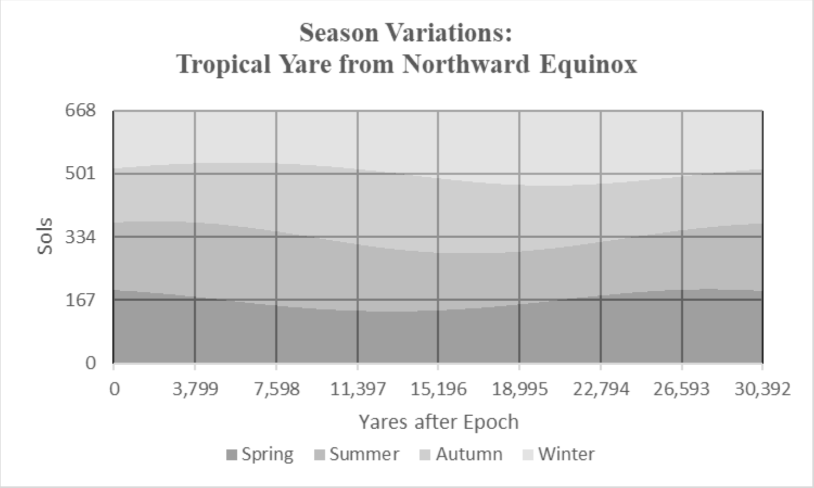


Figure 17: Northern Equinox Mean Tropical Yare Seasonal Variation over Uber-Yare

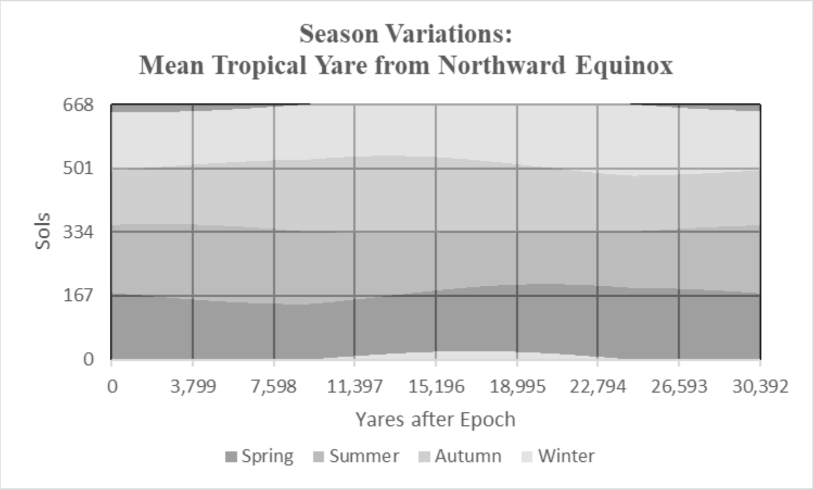


Figure 18: Central Date Dissonance of Northward Equinox Tropical Calendar over Uber-Yare

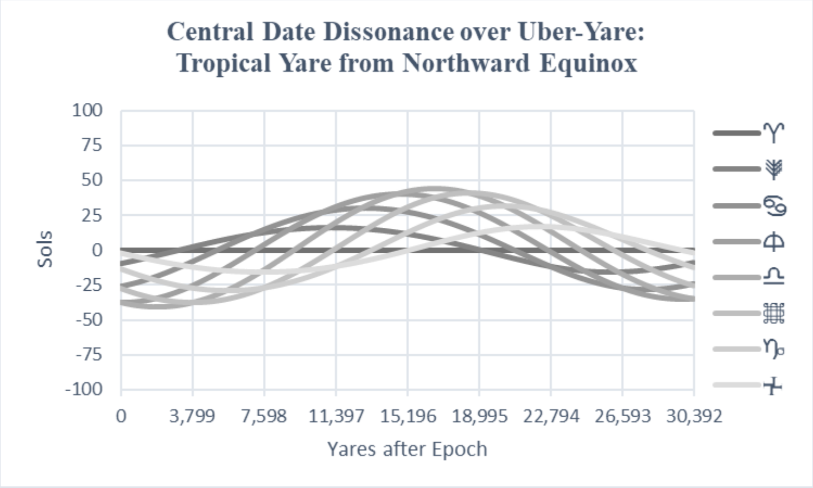


Figure 19: Central Date Dissonance of Southern Solstice Tropical Calendar over Uber-Yare

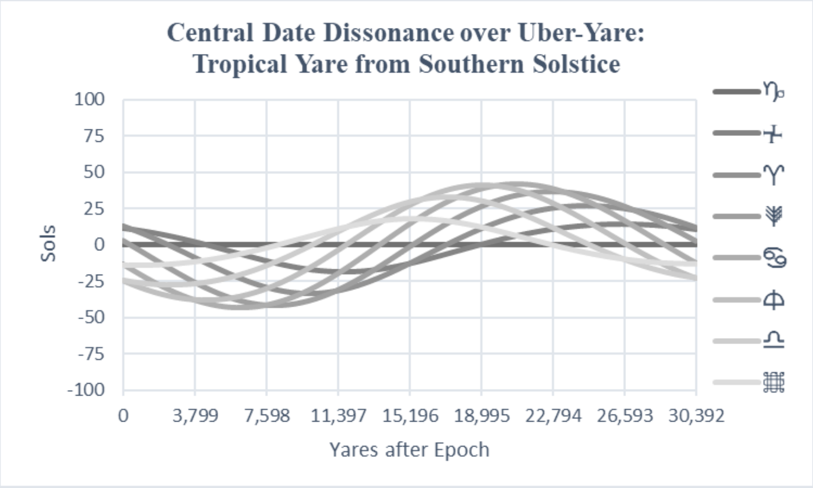


Figure 20: Central Date Dissonance of Northward Equinox Mean Tropical Calendar over Uber-Yare

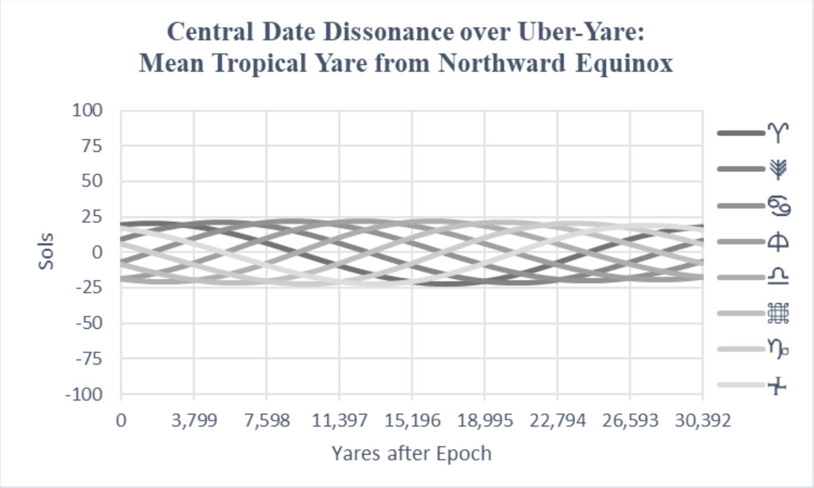


Figure 21: Central Date Dissonance of Southern Solstice Mean Tropical Calendar over Uber-Yare

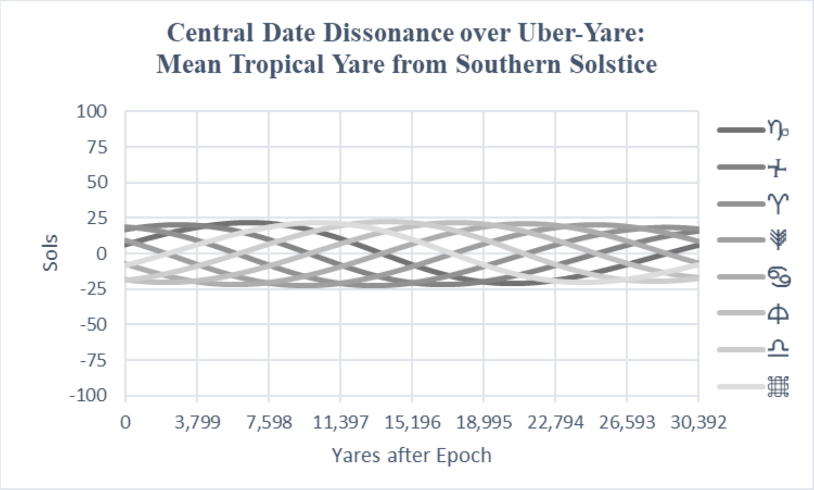
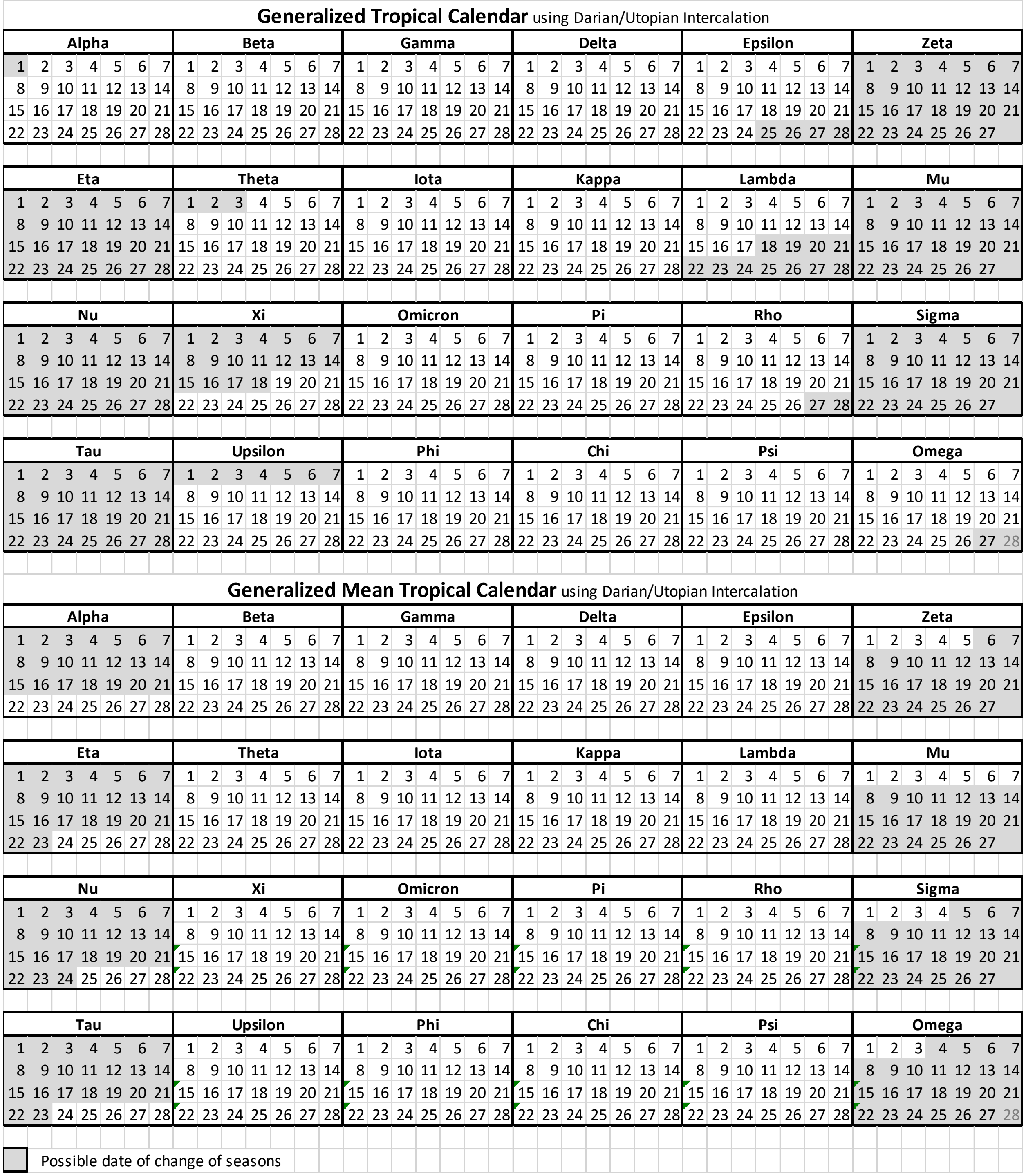


Figure 22: Generalized Calendars Showing Possible Dates of Seasonal Changes over Uber-Yare

The month names substituted here are neutral regarding the calendars under discussion.



Generalized Tropical Calendar using Darian/Utopian Intercalation

Alpha							Beta							Gamma							Delta							Epsilon							Zeta						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14
15	16	17	18	19	20	21	15	16	17	18	19	20	21	15	16	17	18	19	20	21	15	16	17	18	19	20	21	15	16	17	18	19	20	21	15	16	17	18	19	20	21
22	23	24	25	26	27	28	22	23	24	25	26	27	28	22	23	24	25	26	27	28	22	23	24	25	26	27	28	22	23	24	25	26	27	28	22	23	24	25	26	27	

Eta							Theta							Iota							Kappa							Lambda							Mu						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14
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Nu							Xi							Omicron							Pi							Rho							Sigma						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
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Tau							Upsilon							Phi							Chi							Psi							Omega						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
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Generalized Mean Tropical Calendar using Darian/Utopian Intercalation

Alpha							Beta							Gamma							Delta							Epsilon							Zeta						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
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Eta							Theta							Iota							Kappa							Lambda							Mu						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
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Nu							Xi							Omicron							Pi							Rho							Sigma						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
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Tau							Upsilon							Phi							Chi							Psi							Omega						
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
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▨ Possible date of change of seasons

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[i] There is no universally accepted term for a Martian year. ‘Yare’ is used here due to its close spelling and pronunciation to ‘year’. The last four letters of the plural spells ‘Ares’, the Greek name for Mars.

[ii] The precession cycle can be defined as a migration of the perihelion about the tropical frame, much as Mars does in one yare. Since the sun imparts greater light and energy onto Mars at the perihelion, it has a great effect on the climate. Therefore, the position of the perihelion within the tropical frame defines patterns of yarely seasons that can be described as uber-seasons

[iii] The online Merriam-Webster Dictionary (Merriam-Webster, n.d.) defines ‘analemma’ as “a plot or graph of the position of the sun in the sky at a certain time of day (such as noon) at one locale measured throughout the year that has the shape of a figure 8 om Earth the sun never varies from local noon by as much as 17 minutes: from Mars the sun can vary from local noon by more than 50 minutes.

[iv] The Celtic midseason points serve as better seasonal boundaries on Mars than the traditional solstices and equinoxes. On Earth, there is a dual understanding of seasons. The ‘daylight’ seasons involve the duration of daylight; the temperature seasons involve the hot and cold seasons at higher latitudes. These are different due to the ability of Earth’s seas and atmosphere to retain heat. On Mars these are the same, and more aligned with the midseason points. Martian colonists would not perceive the seasons we do, but what might be termed para-seasons separated by the four Celtic points.

[v] There is a very subtle consideration with this approximate equality. In order for the integral to be valid. The equation of date at every mean anomaly must be continuous over time. As defined in this paper it is continuous over time. This is true for points that are, at a given point in time, associated with the tropical markers such as the northward equinox. The ‘northward equinox’ understood as the instant Mars reaches the appropriate point in its orbit occurs only once per tropical yare and cannot be integrated. The ‘northward equinox’ understood as the appropriate point on the orbit regardless of the presence of the planet can be integrated. This paper chooses to use the latter interpretation. Calculating the former interpretation would require a better understanding of eccentricity changes than exists at the writing of this paper. It would involve numerical methods whose results include positive deviations from zero for those points that appear late in the calendar yare when the eccentricity is increasing and negative deviations when the eccentricity in decreasing. These deviations would tend to cancel as the direction of change for the eccentricity will reverse during the uber-yare. Any residual could either be used to shift all dates, as a body, or simply dismissed as an unavoidable change in climate.

[vi] A small fraction of the asymmetry seen about the individual seasonal boundaries is due to the intercalation pattern. Most of the asymmetry is due to the value of the eccentricity at the time when the maximum date dissonance occurs.