THE DARIAN CALENDAR AND INTERCALATION

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ABSTRACT

Over thirty separate proposals for a Mars calendar have been published in some form. Each addresses the problem of a non-integral number of sols (Martian days) in a Martian year. The method universally chosen is to vary the number of sols in a regular pattern over centuries and millennia so that the overall average length of a calendar year closely approximates the physical year. This is called ‘intercalation’.

Many of the calendar proposals have considered intercalation for the current Martian year. However, most do not adequately address the evolution of the length of the year over time. Any useable civil or scientific calendar must periodically adjust its intercalation pattern to compensate for this continuous change.

One of the calendars that does address this issue is the Darian Calendar. It was developed by Thomas Gangale in 1998, and a varying intercalation schema was added in 2006. It is one of the most popular proposals.

This paper examines some issues with the intercalation of the Darian calendar to suggest possible improvements and to serve as an example for other calendar proposals. Intercalation is surprisingly complex; this paper is limited to the mathematics and methods for a basic intercalation design It does not address the sol/week intercalation issue, nor the insertion point of the additional day or days.

INTRODUCTION

Over thirty separate proposals for a Martian calendar have been made over more than a century. At the writing of this paper, none has been adopted. The Darian Calendar, first proposed by Thomas Gangale in 1998 (Gangale, 1998), has become one of the most popular. When first proposed, it did not consider the changing length of the Martian year (termed a ‘yare’ in this paper[i]). This was addressed in 2006 (Gangale, 2006). There were some peculiarities in this addition that represent issues common to many of the proposals. This paper concerns those peculiarities and presents a method of correcting them both in the Darian Calendar and other proposals similarly affected.

ANALYSIS

Intercalation is the process of varying the length of a calendar year to conform, in the long term, to the length of the physical year. The Common Era calendar accomplishes this by using a fairly simple pattern of leap years. Mars' orbit is not as consistent nor as circular as Earth's, so the length of the yare continually changes and requires an extended, multi-pattern intercalation schema.

Many of the proposals for a calendar for Mars have no extended intercalation plan at all. This would cause the weather and daylight duration conditions on Mars to migrate along the calendar over the centuries. Estimates for the time taken to circumnavigate the yare range from about 26,600 to 30,400 yares. This is roughly 50,000 to 57,000 common era years.

The Darian intercalation schema address a 10,000 yare time period from the invention of the telescope. It has five intercalation domains (see Table 4 and Table 5) ranging from 1,600 to 2,800 yares in duration.

There are four features of the Darian intercalation schema that signal possible issues.

1. There was a small numerical oversight in the first domain.
2. The 'divisibility heuristic' of intercalation was not followed in the second, fourth and fifth domains.
3. During a period when the domains should be growing longer, they grow shorter.
4. The fifth domain recommends a yare much longer than is currently possible for Mars

Examination of these four features reveals that there are two minor issues and one major issue. The smallest issue involves the first domain having a yare zero that is not calculated into the average length of a yare that appears in the fifth column. By definition (see Table 4) the yare zero must be a leap yare. That will add a sol to the total during that domain, and a yare to the duration. This has the sum effect of making the whole domain about 0.409 sols longer than indicated.

The second minor issue concerns miscalculating average yares within a domain. For instance, the second domain has the following rule. "(Y-1)\2 + Y\10 - Y\150". The backslash indicates integer division. The rule means that every odd yare is a leap yare, and every tenth yare except those evenly divisible by 150 is a leap yare. Clearly the summation formula **used** was 668 + ½ + 1/10 – 1/150, **not** using integer division. This would be correct if the duration of the domain was divisible by 150, which it is not. The domain begins at 2,001. 2,000 is not divisible by 150, so there is a 50 yare surplus coming into that domain. There will be 19 instances of the '-1/150' rule being applied, but the yare length presented indicates 18.$\underline{6}$ (18.666…) was used to determine the average yare. Consequently, there is a 0.$\underline{6}$-sol error over that domain.

There are similar errors in the fourth and fifth domain. The total error makes the intercalation several days less accurate than indicated in the 2006 paper.

The appropriate rule may be expressed as follows. "The number of yares in an intercalation domain must be evenly divisible by every intercalation denominator it contains." Using this rule assures that the correct number of leap yares can be easily determined, even if a surplus from a previous domain exists. This paper refers to this rule as the 'divisibility rule'. This rule may be violated, but the error produced must be corrected by subsequent domains.

The Cyclic Nature of the Length of the Yare

The major issue is somewhat complicated and will consume the greater proportion of this paper. Gangale states the following in his 2006 paper:

> *"Theoretically, with a mean calendar year of 668.5905 sols, this error amounts to only one sol in 5,000 Martian years; however, the actual error will depend on the changes in Mars' orbital elements, rotational period, and the rate of the precession of the pole vector over this period of time."*

This is only partly true. A major causal factor for the change in any tropical yare is the eccentricity of the orbit, even if no orbital elements change. Certainly, the factors Gangale listed will ultimately be important, particularly the relative precession between the apsides and the equinoxes. He stated in the same paper:

> *"Refinement of the intercalation series will need to await the determination of a value for the second order term for the variation of the Martian vernal equinox year."*

The intercalation pattern can be improved immediately. The issue here is that an instant linear approximation of a cyclic process was used to predict the length of the northward equinox yare over much too long a period. Not considering minor oscillations in parameters other than eccentricity and the relative precession between the apsidal and tropical frames, the length of the yare varies in a predictable manner.

Figure 1 shows the orbit of Mars in terms of its eccentric orbit. Since this involves the two apsides, perihelion and aphelion, it will be termed here the 'apsidal frame'.

Figure 2 shows the orbit of Mars in terms of the axial tilt. Since this determines the tropical points, it will be termed here the 'tropical frame'. The orbital longitude is an angle measured from the sun to some point on the tropical frame whose zero is the vernal equinox, the start of northern spring.

The passage of Mars from any orbital longitude on the tropical frame back to that same orbital longitude is a tropical yare. As explained below, the length of a tropical yare depends on the orbital longitude chosen. The average of all possible choices is the mean tropical yare.

These two frames both precess, the apsidal frame in a prograde manner relative to Mars and the tropical frame in retrograde. It is the angular relationship between these two frames that defines all one needs to know about the calendar. The tropical frame determines where the tropical points are, and the apsidal frame determines when Mars arrives at those points. The change in the angle between these frames determines the changes imposed on the calendar over time.

Figure 3 shows how the apsidal frame affects the velocity of Mars during an apsidal yare. The true anomaly (θ: theta) is the angle of some point on the orbit (measured from the sun) whose zero is at the perihelion. The mean anomaly (M: MU) is a mathematical fiction that represents where Mars would be if it had a circular orbit. This difference is terribly important to a calendar. Knowing only the length of the apsidal yare, one can use this geometry to determine how many sols any point on the orbit has varied from its mean place within the tropical yare. This paper refers to this equation as the 'equation of date' since it is closely related to the 'equation of time' used to describe the analemma[ii].

The passage of Mars from any true anomaly on the apsidal frame back to that same true anomaly is an apsidal or anomalistic yare. The length of an apsidal yare generally does not depend on the true anomaly chosen. Apsidal yares may vary due to a combination of the true anomaly and changes to the eccentricity, but for the purposes of this paper there is only one apsidal yare.

The constants used in equations are defined in Table 3. They were derived from a paper by Michael Allison. (Allison, 1997).

Equation 1: Equation of Date

M (MU) is the mean anomaly. θ (theta) is the true anomaly.

The following equation set represents the 'equation' of date. It can be used to find M from θ. It is necessary to estimate and iterate to find θ from M.

E (EPSILON) is called the eccentric anomaly. Δ (DELTA) is the difference between the mean and true anomaly, sometimes called the 'equation of center'. Y_A is the length of the apsidal yare. O (OMICRON) is the 'equation of date'.

$$E = sin^{-1}\left(\frac{\sqrt{1-e^2}\sin(\theta)}{1+e\cos(\theta)}\right)$$

$$M = E - e\sin(E)$$

$$\Delta = (\theta - M)$$

$$O = \frac{Y_A\,\Delta}{2\,\pi}$$

The current values of the equation of date at each point on Mars' current orbit is charted in Figure 4. The symbols presented on the chart represent the four standard seasonal boundaries and four mid-season markers. These symbols are defined in Table 1. To avoid hemisphere bias, the vernal equinox is now sometimes called the northward equinox, and the other three boundaries are similarly renamed. This paper uses the new convention. This paper also uses the Celtic names for the four mid-season markers because of their convenient definition and available symbols.[iii] Lower case is used to distinguish them from the Celtic holidays. They are defined in Table 2.

In this chart, one can see that the southern solstice occurs about 29 sols after the perihelion and about six sols after its mean. The northward equinox occurs about 183 sols after the perihelion and 19 sols after its mean. Over the course of one precession cycle (termed an uber-yare in this paper)[iv], the relative precession moves each tropical marker to the left. Eventually each marker will take on each of the values on the curve. The result of this movement is that the length of a tropical yare using any of these markers constantly changes.

The length of the yare can be determined by adding the first derivative of the equation of date to the mean tropical yare.

Equation 2: Length of Tropical Yares by True Anomaly

> E'_θ is the first derivative of E in terms of Θ. M'_θ is the first derivative of M in terms of θ. Δ'_θ is the first derivative of Δ in terms of θ. Y_m is the length of the mean tropical yare. U_C is the number of apsidal yares in a precession cycle. Y_θ is the length of the specific tropical yare at θ.

$$E'_\theta = \frac{\sqrt{1 - e^2}}{1 + e\cos(\theta)}$$

$$M'_\theta = E'_\theta(1 - e\cos(E))$$

$$\Delta'_\theta = (1 - M'_\theta)$$

$$Y_\theta = Y_m + \frac{\Delta'_\theta Y_A}{U_C}$$

The current yare lengths are charted in Figure 5. The tropical markers are also precessing to the left on this chart. The southern solstice is approaching the perihelion, where it will experience the longest yare. The northward equinox is approaching a point where it will experience a yare equal to the mean tropical yare. Obviously, the rate at which the length of a yare for any marker changes. This means that the rate at which the intercalation must change varies over time. While this yare is currently getting longer for the northward equinox, it will soon be doing so at a decreasing rate. It should reach a maximum in about 9,554 yares at 668.59634 sols. This is far short of the 668.59833 presented in the 2006 paper.

The intercalation change rate is the first derivative of the length of the yare, or the second derivative of the equation of date.

Equation 3: Tropical Yare Intercalation Change Rate by True Anomaly (per Millennia)

E''_θ is the second derivative of the eccentric anomaly in terms of θ. M''_θ is the second derivative of the mean anomaly in terms of θ. Δ''_θ is the second derivative of Δ in terms of θ. Y'_θ is the first derivative of the length of the yare; the rate of changed of intercalation.

$$E''_\theta = \frac{E'_\theta e \sin(\theta)}{1 + e\cos(\theta)}$$

$$M''_\theta = E''_\theta(1 - e\cos(E)) + {E'_\theta}^2 e \sin(E)$$

$$\Delta''_\theta = (-M'_\theta)$$

$$Y'_\theta = \frac{2000\,\pi\ \Delta''_\theta\, Y_A}{{U_C}^2}$$

The current intercalation change rates are also charted in Figure 5. Since the seasonal markers are moving right to left, the change rate, which is the first derivative of the yare length, is inverted from what one might expect. Note that the southern solstice is undergoing very little change in the length of its yare as it passes the perihelion. It is important to note that the intercalation change rate for the northward equinox will have gone to zero near this point. The zero of the change rates would be exactly at the perihelion if it were not for the eccentricity increasing at this time. This effect is subtle because change rates are relatively insensitive to eccentricity changes near the apsides. See Figure 6.

Equation 4: Tropical Yare Sensitivity to Eccentricity Changes by True Anomaly

E'_e is the first derivative of the eccentric anomaly in terms of eccentricity. M'_e is the first derivative of the mean anomaly in terms of e. S_e is the sensitivity of Y_θ in terms of the eccentricity.

$$E'_e = \frac{-\sin(\theta)}{|\cos(\theta)\,e + 1|\,\sqrt{1 - e^2}}$$

$$\boldsymbol{M'_e = E'_e - \frac{\sin(\theta)(1 - e^2(\cos(\theta)\,e + 2))}{(\cos(\theta)\,e + 1)^2\sqrt{1 - e^2}}}$$

$$S_e = \frac{0.001\, M'_e Y_A}{2\,\pi}$$

Intercalation and the Cyclic Yare Length

Considering the above description of the length of the yare over time, it is possible to discuss a major issue within the current Darian calendar intercalation.

Gangale is certainly correct that better measurement of Mars' orbital elements will improve the accuracy of intercalation. The length of an uber-yare is very sensitive to changes in either the apsidal or tropical precession. This paper assumes an uber-yare of 30,391 yares. This matches the time period used in Gangale's 2006 paper.

Figure 7 and Figure 8 compare the current Darian intercalation schema to the actual yare length over the two extremes of the uber-yare estimates.

In Figure 7, the northward equinox is passing through the maximum increase in yare length. In several thousand yares the northward equinox will have passed the maximum and by the end of the horizon it will be undergoing very little change. Since the maximum length of an intercalation domain is approximately the inverse of the mean intercalation change, one would expect the domain length to increase from the change peak to the horizon. This does not happen with the current intercalation schema. In addition, the length of the yare during the last defined domain is much longer than the longest possible yare, even when Mars reaches its maximum eccentricity in about 13,000 yares. If one assumes the lower end of the uber-yare duration, the overshoot of the intercalation is even more pronounced.

Table 6 shows a possible replacement for the current intercalation schema. This is referred to in this paper as the 'adjusted' schema. This schema assumes an uber-yare of about 30,400 yares, but a very similar schema can be produced for any uber-yare duration. This schema was generated by finding the domain with the longest possible duration that stays within one half sol of the actual yare length. Obviously, this will be centered near the perihelion. This calculation observes only the average yare of the domain and ignores the shorter cycles (such as the 10-yare oscillation) that introduce short-term oscillations into the error. Ultimately other constraints from surrounding domains and the asymmetric curve of the yare length pushed this domain to error ranges from -0.68 to +0.29. This is still within a narrow band. Then the two domains before and after were designed to match the error range of the longest domain. The earliest domain was then created with the same criteria. The total error range of the schema is from -0.68 to +0.41 over 17,200 yares. More sophisticated techniques and automation should produce an even better answer.

Figure 9 compares the adjusted intercalation schema to the actual length of the yare over 17,200 yares. Note that the duration of the intercalation domains is increasing until it passes the perihelion. Since the mean tropical yare is 668.5922 sols long, this chart indicates that all the yares longer than that are encompassed by only three domains. Therefore, it should take only eight domains to span the entire uber-yare.

It is interesting to note that either solstice is about 2,400 yares into an 8,000 yare domain. A calendar with a 6,400 yare horizon and **no** intercalation changes could be produced by using a solstice to determine the start of the yare.

Figure 10 charts the accuracy of Gangale's intercalation schema against the two extremes of the uber-yare estimate. It also shows the adjusted schema's accuracy over a much longer period. This graph, once again, ignores any shorter-term error oscillations introduced by cycles within the domain.

Figure 11 presents the root mean square errors of both Gangale comparisons and the adjusted comparison over their respective horizons. The large number for the 26,600 yare uber-yare estimate is clearly caused by an expectation of a longer uber-yare. In a complex solar system, none of these RMSEs will be achieved, but the relative proportions will likely be maintained.

It is worth noting that the Utopian Calendar (Moss, 2000), another proposal of note, uses the same intercalation pattern and suffers from the same issues.

CONCLUSION

Until more accurate measurements of Mars' orbital elements, it might be appropriate to specify intercalation changes in terms of a true anomaly or an orbital longitude than in terms of yares or years.

The equations in this paper are more accurate over time than a low-ordered polynomial approximation. Any new proposal for a Martian calendar, or new intercalation schema, will have to consider both these equations and some form of the divisibility rule.

It appears that an optimal intercalation schema can be derived using as few as eight domains to span an entire uber-yare. These domains will have to be modified over time as our knowledge of Mars' orbit and perturbations increase. This schema is only slightly different from the adjusted schema presented here. For example, such a schema would use the mean tropical yare as the yare length twice an uber-yare.

FURTHER RESEARCH

It appears that an algorithm can be developed to produce an optimal intercalation schema for any planet. This algorithm may be addressed in a future paper.

Intercalation is quite a bit more involved than appears here, including the efficacy of having built-in changes at all. Pinning the beginning of the yare to any tropical marker has unfortunate side-effects. Figure 4 shows how the northward equinox has strayed from its mean position since it last passed the aphelion. Note that the southward equinox is about as far after its mean date as the northward equinox is before its mean date. Over the millennia, the southward equinox will move ahead about 40 days while the start of the yare will move backward 40. That means that the southward equinox will slide 80 days across the calendar. This is an anathema to the reason for pinning the first day of the yare to a tropical marker, and contrary to the purpose of a calendar itself.

Figure 12 shows how the Martian seasons will oscillate over the uber-yare. These oscillations can be dampened by using the equation of date for the calendar over an uber-yare in the same manner as the equation of time acts for the clock over the course of a yare. See Figure 13. The northward equinox would oscillate about its mean anomaly in its own 'uber-analemma'. Making this change would mean converting the Darian calendar to a mean tropical calendar. While intercalation changes due to the seemingly chaotic interplay of gravitational fields within the solar system would still be necessary, no built-in changes would be required. This conversion would also minimize the difference between every calendar date and its expected seasonal properties, which is arguably the purpose of a calendar.

These issues will be addressed in a future paper.

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APPENMDIX A: TABLES

Table 1: Chart Symbols

Tropical Marker Symbols		
Symbol	**Meaning**	**Solar Longitude (λ)**
♈	northward equinox	0°
[illegible]	beltain	45°
♋	northern solstice	90°
[illegible]	lammas	135°
♎	southward equinox	180°
[illegible]	samhain	225°
♑	southern solstice	270°
[illegible]	imbolic	315°
Apsidal Marker Symbols		
Symbol	**Meaning**	**Solar Longitude (θ)**
P	perihelion	0°
A	aphelion	90°

Table 2: Non-Standard Term Definitions

Non-Standard Terms	
Term	**Meaning**
yare	Martian year (alternative: mir)
beltain	45° from the northward equinox
lammas	45° from the northern solstice
imbolic	45° from the southward equinox
samhain	45° from the southern solstice
uber-yare	One cycle of the relative precession between the apsidal and tropical frames, often called the precession cycle

Table 3: Constants Used in Equations

Equation Constants	
Element	**Value**
sols in an apsidal yare	668.61410
sols in a mean tropical yare c2000	668.59210
apsidal yares in an uber-yare (precession cycle)	30,391.55000
northward equinox true anomaly c2000	1.90276

Table 4: Darian Intercalation Rules

Darian Intercalation Rules			
Start	End	Duration	Leap Yare Rules
0	2000	2001	(Y-1)\2 + Y\10 – Y\100 +Y\100
2001	4800	2800	(Y-1)\2 + Y\10 – Y\150
4801	6800	2000	(Y-1)\2 + Y\10 – Y\200
6801	8400	1600	(Y-1)\2 + Y\10 – Y\300
8401	10000	1600	(Y-1)\2 + Y\10 – Y\600

Table 5: Darian Intercalation and Divisibility

Darian Intercalation and Divisibility							
Start	End	Duration	Gangale		Actual		Error
			Sols+	Yare	Sols+	Yare	
0	2000	2001	1182	668.59100	1183	668.59120	0.40900
2001	4800	2800	1661.33333	668.59333	1662	668.59357	0.66667
4801	6800	2000	1190	668.59500	1190	668.59500	0
6801	8400	1600	954.66667	668.59667	955	668.59688	0.33333
8401	10000	1600	957.33333	668.59883	958	668.59875	0.66667
		10001	**5945.33333**	**668.59453**	**5948**	**668.59474**	**2.07213**

Table 6: Adjusted Darian Intercalation

Adjusted Darian Intercalation				
Start	End	Duration	Expected	
			Sols+	Yare
1	2800	2800	1656	668.59143
2801	5800	3000	1782	668.594
5801	14050	8250	4917	668.604
14051	17250	3200	1900	668.59375
		10000	**17250**	**668.59449**

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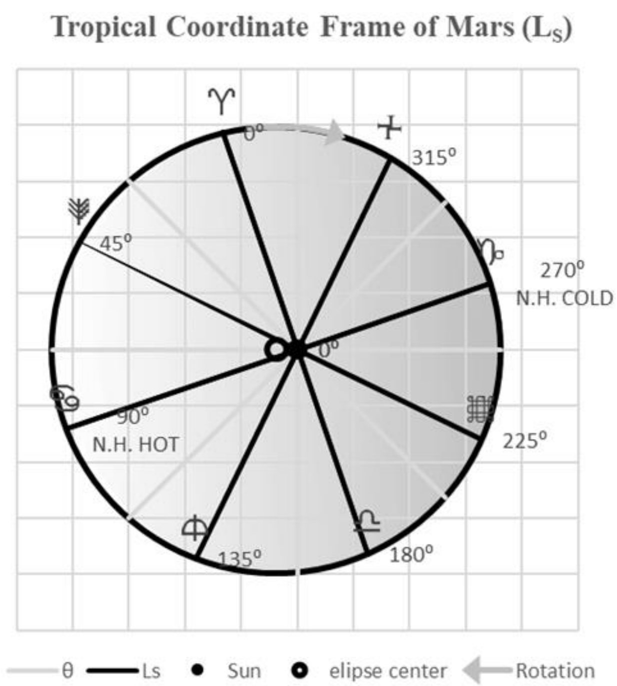
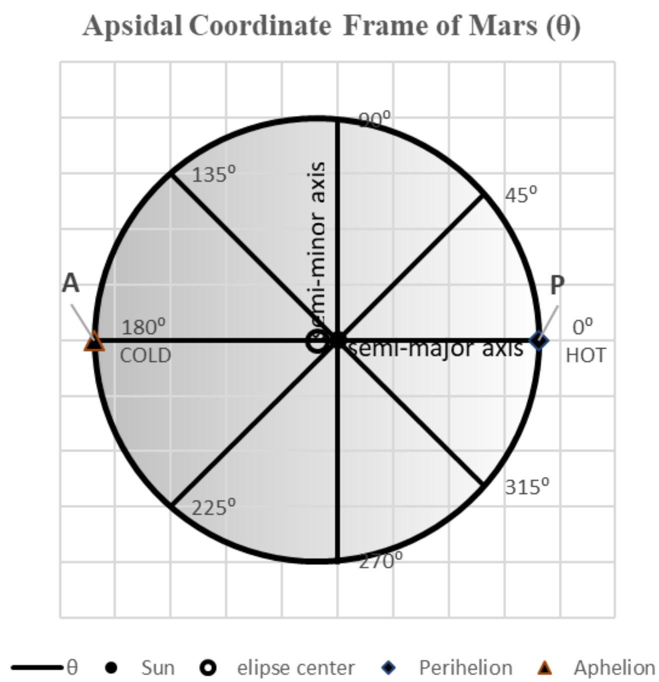
APPENDIX B: FIGURES

Figure 1: Tropical Coordinate Frame



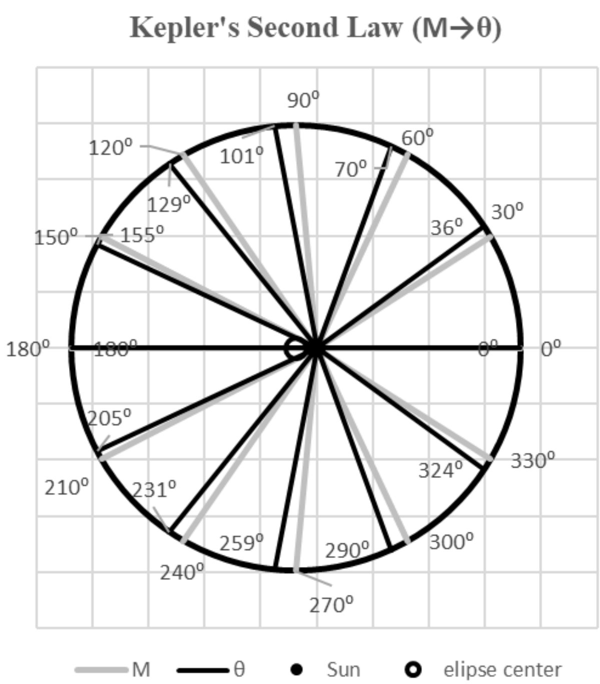
This tropical coordinate frame measures an angle from the northward (vernal) equinox in the direction the planet revolves. This angle is called the solar longitude and is denoted by L_S.
The shading indicates the heat that impacts the northern hemisphere.

Figure 2: Apsidal (Anomalistic Coordinate Frame



The apsidal coordinate frame measures an angle from Mars' nearest approach (perihelion) in the direction Mars revolves. This angle is called the true anomaly or just anomaly and is usually denoted θ (theta) or occasionally ν (nu).
Eccentricity (e) is the distance from the sun to the elliopse center divided by the semi-major axis.
The shading indicates the intewnsityof the solar constant.

Figure 3: Kepler's Second Law and the Mean Anomaly



Kepler's second law states that a line from the sun to a planet sweeps out equal areas in equal times. Each heavy black line (θ) indicates how far Mars will travel in one eighth of a yare.
The mean anomaly is denoted by M (mu) and indicates where Mars would be if its orbit was circular.

Figure 4: Equation of Date

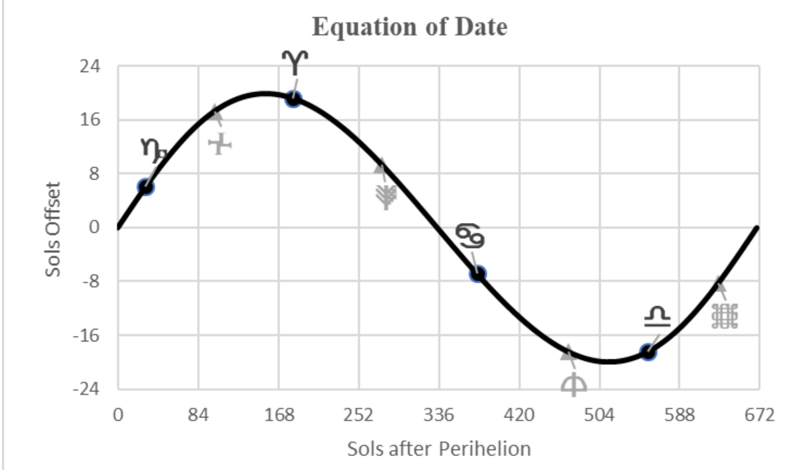


Figure 5: Current Yare Lengths and Intercalation Rates

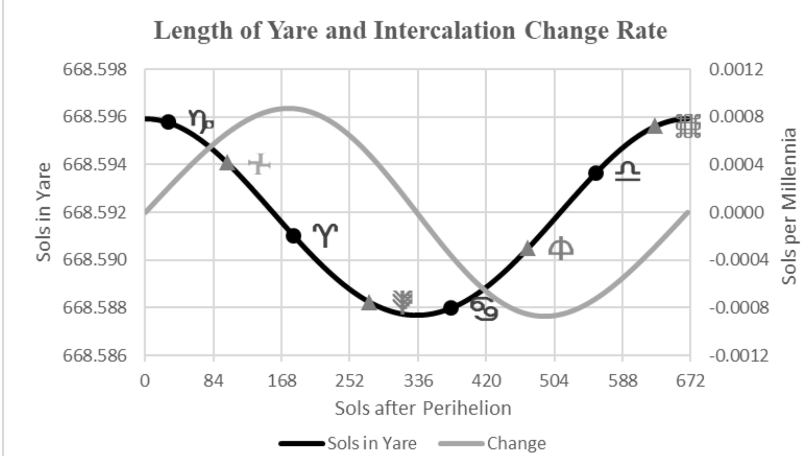


Figure 6: Sensitivity to Eccentricity Changes

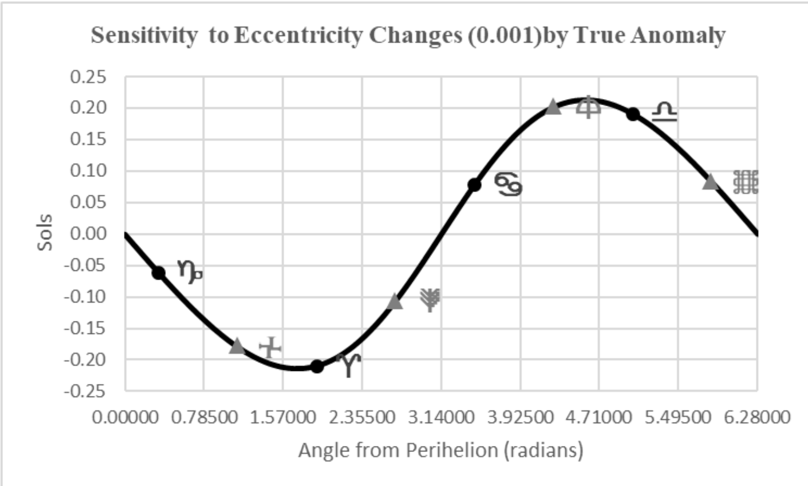


Figure 7: Length of Yare and Intercalation (30.4k Cycle)
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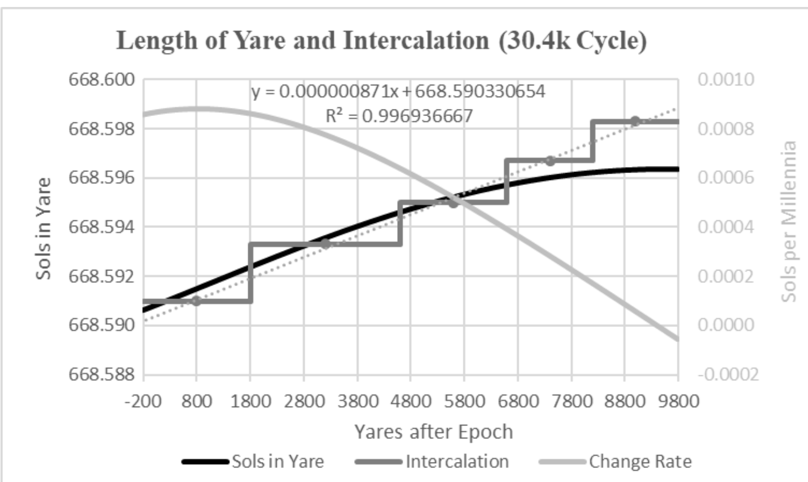


Figure 8: Length of Yare and Intercalation (26.6k Cycle)

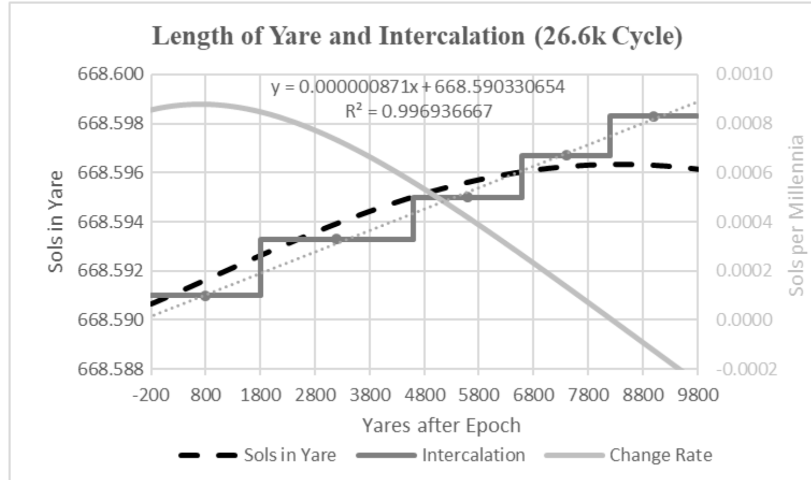


Figure 9: Adjusted Length of Yare and Intercalation (30.4k Cycle)

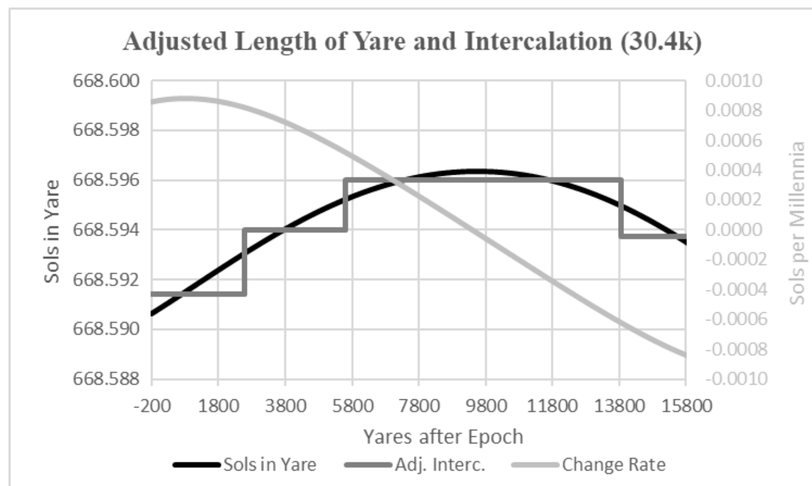


Figure 10: Intercalation Accuracy

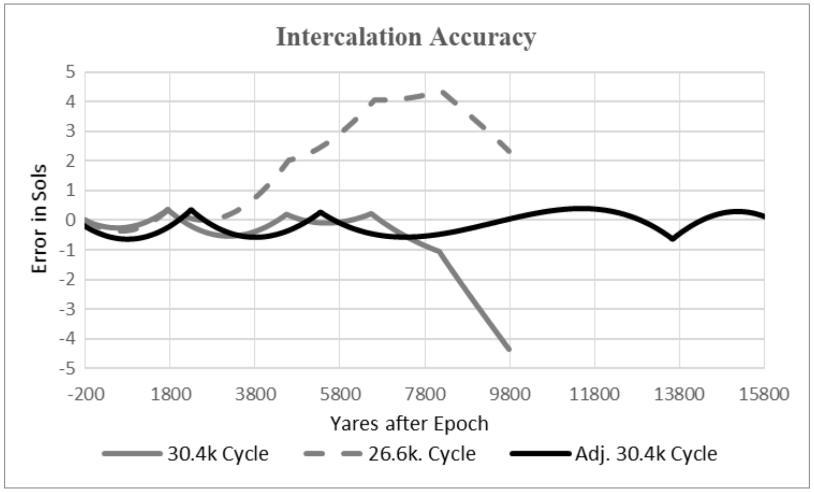


Figure 11: Intercalation Root Mean Square Error

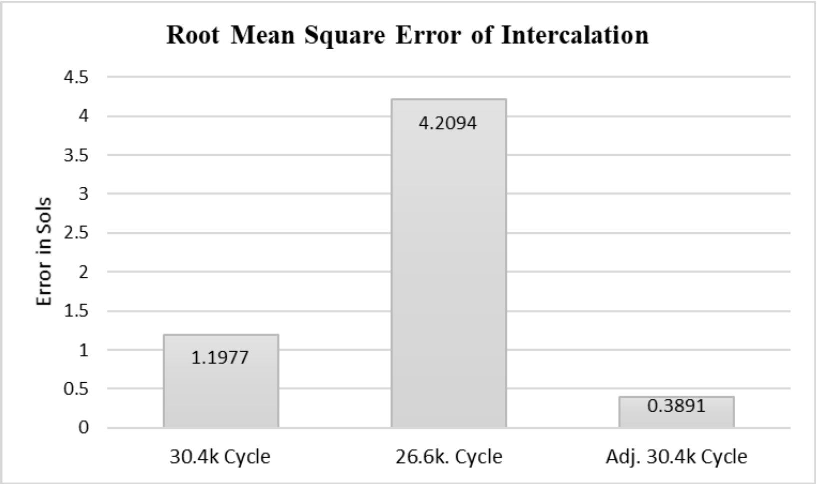


Figure 12: Seasonal Oscillations Using Tropical Yare

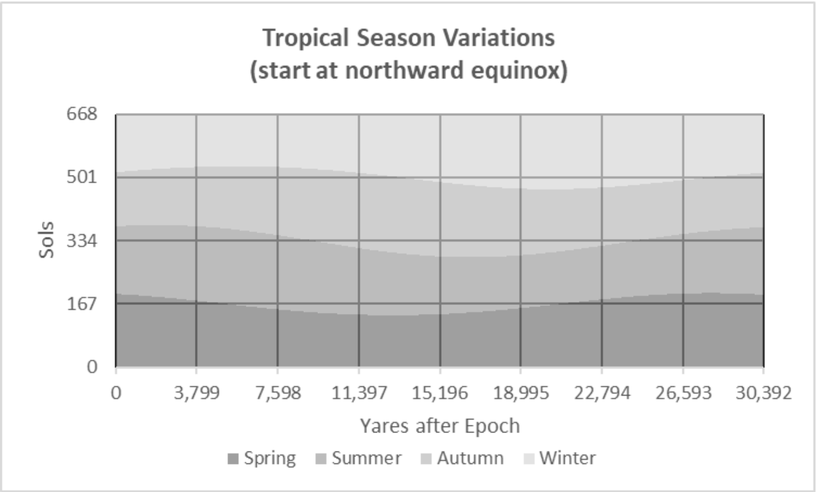
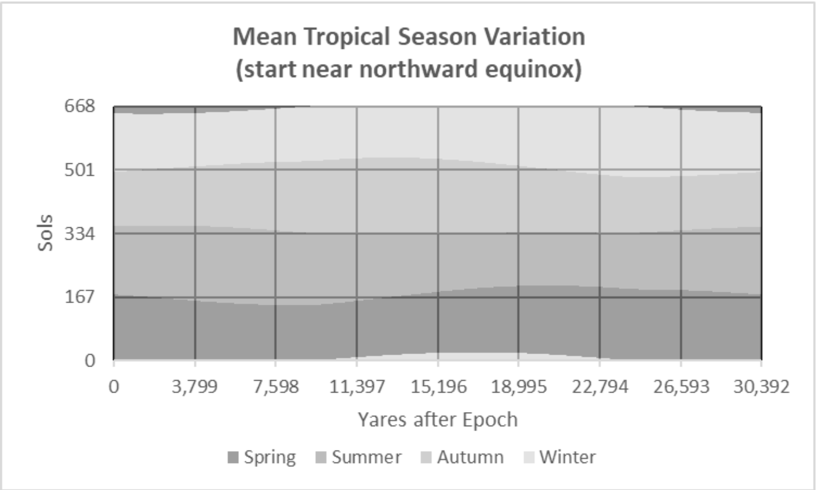


Figure 13: Seasonal Oscillations Using Mean Tropical Yare



[i] There is no universally accepted term for a Martian year. 'Yare' is used here due to its close spelling and pronunciation to 'year'. The last four letters of the plural spells 'Ares', the Greek name for Mars.

[ii] The online Oxford Living Dictionary (US) (Oxford Dictionaries, n.d.) defines 'analemma' as "The asymmetrical figure-of-eight curve that can be traced in the sky at a given place showing the position of the sun at mean solar noon on successive days of a year." On Mars the analemma is tear drop, not figure-of-eight, shaped and quite a bit wider than on Earth. From Earth the sun never varies from local noon by as much as 17 minutes: from Mars the sun can vary from local noon by more than 50 minutes.

[iii] The Celtic midseason points serve as better seasonal boundaries on Mars than the traditional solstices and equinoxes. On Earth, there is a dual understanding of seasons. The 'daylight' seasons involve the duration of daylight; the temperature seasons involve the hot and cold seasons at higher latitudes. These are different due to the ability of Earth's seas and atmosphere to retain heat. On Mars these are the same, and more aligned with the midseason points. Martian colonists would not perceive the seasons we do, but what might be termed para-seasons separated by the four Celtic points/

[iv] The precession cycle can be defined as a migration of the perihelion about the tropical frame, much as Mars does in one yare. Since the sun imparts greater light and energy onto Mars at the perihelion, it has a great effect on the climate. Therefore, the position of the perihelion within the tropical frame defines patterns of yarely seasons that can be described as uber-seasons.